



Broj: 02/1-176/1
Datum: 15.02.2021

UNIVERZITET CRNE GORE

- Centru za doktorske studije -

- Senatu -

O V D J E

U prilogu dostavljamo Odluku Vijeća Elektrotehničkog fakulteta sa sjednice od 11.02.2021. godine i obrazac **D2**, sa pratećom dokumentacijom, za kandidata MSc **Stefana Vujovića**, na dalji postupak.



DEKAN, -a
N. Perović - Bugarić
Prof. dr Saša Mujović



ISPUNJENOST USLOVA DOKTORANDA

OPŠTI PODACI O DOKTORANDU			
Titula, ime, ime roditelja, prezime	MSc Stefan Zoran Vujović		
Fakultet	Elektrotehnički fakultet		
Studijski program	Doktorske studije elektrotehnike		
Broj indeksa	5/13		
NAZIV DOKTORSKE DISERTACIJE			
Na službenom jeziku	Analiza, implementacija i primjena gradijentnih algoritama za rekonstrukciju kompresivno odabranih signala		
Na engleskom jeziku	Analysis, implementation and applications of gradient based algorithms for reconstruction of compressively sampled signals		
Naučna oblast	Elektrotehnika / Obrada signala		
MENTOR/MENTORI			
Prvi mentor	Prof. dr Miloš Daković	Elektrotehnički fakultet, Univerzitet Crne Gore, Podgorica, Crna Gora	Elektrotehnika/ Obrada signala
KOMISIJA ZA PREGLED I OCJENU DOKTORSKE DISERTACIJE			
	Prof. dr Irena Orović	Elektrotehnički fakultet, Univerzitet Crne Gore, Podgorica, Crna Gora	Elektrotehnika/ Obrada signala
	Prof. dr Miloš Daković	Elektrotehnički fakultet, Univerzitet Crne Gore, Podgorica, Crna Gora	Elektrotehnika/ Obrada signala
	Doc. dr Jonatan Lerga	Tehnički fakultet, Sveučilište u Rijeci, Rijeka, Hrvatska	Elektrotehnika/ Računarstvo
Datum značajni za ocjenu doktorske disertacije			
Sjednica Senata na kojoj je data saglasnost na ocjenu teme i kandidata	17.5.2019.		
Dostavljanja doktorske disertacije organizacionoj jedinici i saglasnost mentora	1.2.2021.		

Sjednica Vijeća organizacione jedinice na kojoj je dat prijedlog za imenovanje komisija za pregled i ocjenu doktorske disertacije	11.02.2021.
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ISPUNJENOST USLOVA DOKTORANDA

U skladu sa članom 38 pravila doktorskih studija kandidat je dio sopstvenih istraživanja vezanih za doktorsku disertaciju publikovao u časopisu sa **(SCI/SCIE)** liste kao prvi autor.

Spisak radova doktoranda iz oblasti doktorskih studija koje je publikovao u časopisima sa SCI/SCIE liste:

[1] **S. Vujović**, A. Draganić, M. Lakičević, I. Orović, M. Daković, M. Beko, and S. Stanković, "Sparse Analyzer Tool for Biomedical Signals," *Sensors*, 20(9), 2602, doi: 10.3390/s20092602, 2020

Link na rad:

<https://www.mdpi.com/1424-8220/20/9/2602>

Informacija o IMPACT faktoru časopisa:

<https://www.mdpi.com/journal/sensors>

[2] LJ. Stanković, M. Daković, I. Stanković, and **S. Vujović**, "On the Errors in Randomly Sampled Nonsparse Signals Reconstructed with a Sparsity Assumption," *IEEE Geoscience and Remote Sensing Letters*, Vol: 14, Issue: 12, Dec. 2017, pp. 2453 - 2456, DOI: 10.1109/LGRS.2017.2768664

Link na rad:

<https://ieeexplore.ieee.org/abstract/document/8110831>

Informacija o IMPACT faktoru časopisa:

<https://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=8859>

[3] LJ. Stanković, M. Daković, and **S. Vujović**, "Reconstruction of Sparse Signals in Impulsive Disturbance Environments," *Circuits, Systems and Signal Processing*, vol. 2016. pp. 1-28, DOI: 10.1007/s00034-016-0334-3, ISSN: 0278-081X print, 1531-5878 online (on-line published version available on <https://rdcu.be/4XQ8>)

Link na rad:

<https://link.springer.com/article/10.1007/s00034-016-0334-3>

Informacija o IMPACT faktoru časopisa:

<https://www.springer.com/journal/34>

[4] LJ. Stanković, M. Daković, and **S. Vujović**, "Adaptive Variable Step Algorithm for Missing Samples Recovery in Sparse Signals," *IET Signal Processing*, vol. 8, no. 3, pp. 246 -256, 2014. (arXiv:1309.5749v1).

Link na rad:

<https://digital-library.theiet.org/content/journals/10.1049/iet-spr.2013.0385>

Informacija o IMPACT faktoru časopisa:

<https://digital-library.theiet.org/content/journals/iet-spr;jsessionid=cqvpv362anb5.x-iet-live-01>

Međunarodne konferencije:

- [5] M. Brajović, **S. Vujović**, I. Orović, and S. Stanković, "Coefficient Tresholding in the Gradient Reconstruction Algorithm for Signals Sparse in the Hermite Transform Basis," *Applications of Intelligent Systems 2018 (APPIS 2018)*, Las Palmas De Gran Canaria, 8-12 January 2018
- [6] S. Stanković, **S. Vujović**, I. Orović, M. Daković, and LJ. Stanković, "Combination of Gradient Based and Single Iteration Reconstruction Algorithms for Sparse Signals," *17th IEEE International Conference on Smart Technologies, IEEE EUROCON 2017*
- [7] **S. Vujović**, I. Stanković, M. Daković, and LJ. Stanković, "Comparison of a Gradient-Based and LASSO (ISTA) Algorithm for Sparse Signal Reconstruction," *5th Mediterranean Conference on Embedded Computing MECO 2016*, Bar, June 2016
- [8] **S. Vujović**, M. Daković, I. Orović, and S. Stanković, "An Architecture for Hardware Realization of Compressive Sensing Gradient Algorithm," *4th Mediterranean Conference on Embedded Computing, MECO - 2015*
- [9] **S. Vujović**, M. Daković, and LJ. Stanković, "Comparison of the L1-magic and the Gradient Algorithm for Sparse Signals Reconstruction," *22nd Telecommunications Forum , TELFOR, 2014*

[10] LJ. Stanković, M. Daković, and S. Vujović, "Concentration measures with an adaptive algorithm for processing sparse signals," *ISPA 2013*, Trieste, Italy, 4-6 September 2013, pp. 418-423

Obrazloženje mentora o korišćenju doktorske disertacije u publikovanim radovima

Rezultati istraživanja doktoranda Msc Stefana Vujovića koji su korišćeni pri izradi doktorske disertacije su prezentovani kroz 4 rada, publikovana u renomiranih međunarodnim časopisima sa impakt faktorima: **3.27**, **3.83**, **1.68**, **1.69**. Na jednom od pomenuta 4 rada (impakt faktor **3.27**), kandidat je prvi autor. Pored pomenuta četiri rada, dio rezultata je prezentovan i na 6 međunarodnih konferencija.

U radu „Sparse Analyzer Tool for Biomedical Signals” objavljenom u časopisu „Sensors”(impakt faktor **3.27**), izdavača MDPI na kojem je kandidat prvi autor, je predstavljen virtuelni instrument za analizu rekonstrukcionih algoritama kompresivno odabranih signala. Jedan od algoritama koji je implementiran i čije se funkcionalnosti mogu ispitivati kroz instrument je i gradijentni algoritam koji zauzima centralno mjesto u doktorskoj disertaciji. U četvrtoj glavi doktorske disertacije, gdje se analiziraju primjene gradijentnih algoritama, predstavljeni su rezultati primjene na medicinskim signalima kao što su rendgen snimci i ECG signali, a koji su i analizirani u okviru navedenog rada.

Gradijentni algoritam koji je centralna tema doktorske disertacije je predstavljen u radu „Adaptive Variable Step Algorithm for Missing Samples Recovery in Sparse Signals” objavljenom u časopisu *IET Signal Processing* (impakt faktor **1.69**). Predstavljeni algoritam rekonstrukciju kompresivno odabranih signala vrši u vremenskom domenu, za razliku od većine *state-of-the-art* algoritama koji rekonstrukciju vrše u domenu rijetkosti. Rad je citiran preko 80 puta (Google Scholar). Gradijetni algoritmi, tj. njihova analiza, implementacije i primjene se prožimaju kroz drugu, treću i četvrtu glavu doktorske teze.

Primjena gradijenta mjere je takođe korišćena za detekciju šuma. Rezultati ovog istraživanja su predstavljeni u radu „Reconstruction of Sparse Signals in Impulsive Disturbance Environments,” objavljenom u časopisu *Circuits, Systems and Signal Processing* (impakt faktor **1.68**) renomiranog izdavača Springer. Dobijeni rezultati su prikazani u četvrtoj glavi doktorske disertacije.

U tezi je predstavljen i originalni teorijski doprinos u oblasti kompresivnog odabiranja, koji se ogleda u egzaktnoj formuli za grešku u rekonstruisanim koeficijentima signala koji nijesu rijetki, ili su približno rijetki, pri čemu je rekonstrukcija vršena sa pretpostavkom rijetkosti. Rezultati ovog istraživanja su prezentovani u radu „On the Errors in Randomly Sampled Nonsparse Signals Reconstructed with a Sparsity Assumption,” objavljenom u časopisu IEEE Geoscience and Remote Sensing Letters (impakt faktor **3.83**), a kojeg izdaje najprestižniji izdavač za oblast elektrotehnike IEEE. Rezultati predstavljeni u ovom radu su prikazani u prvom poglavlju doktorske disertacije u kojoj su se razmatrali teorijski koncepti kompresivnog odabiranja.

Jedan dio rezultata kandidata je prezentovan na međunarodnim konferencijama među kojima treba istaći EUROCON, ISPA i MECO konferencije.

Datum i ovjera (pečat i potpis odgovorne osobe)

U Podgorici,
11.02.2021.



29 DEKAN-a
V. Popović - Bugarm

Prilog dokumenta sadrži:

1. Potvrdu o predaji doktorske disertacije organizacionoj jedinici
2. Odluku o imenovanju komisije za pregled i ocjenu doktorske disertacije
3. Kopiju rada publikovanog u časopisu sa odgovarajuće liste
4. Biografiju i bibliografiju kandidata
5. Biografiju i bibliografiju članova komisije za pregled i ocjenu doktorske disertacije sa potvrdom o izboru u odgovarajuće akademsko zvanje i potvrdom da barem jedan član komisije nije u radnom odnosu na Univerzitetu Crne Gore



Broj: 02/1-112/2
Datum: 02.02.2021

Na osnovu službene evidencije i dokumentacije Elektrotehničkog fakulteta u Podgorici, izdaje se

P O T V R D A

MSc **Stefan Vujović**, student doktorskih studija na Elektrotehničkom fakultetu u Podgorici, dana 01.02.2021. godine dostavio je ovom Fakultetu doktorsku disertaciju pod nazivom: „**Analiza, implementacija i primjena gradijentnih algoritama za rekonstrukciju kompresivno odabranih signala**“, na dalji postupak.

 **DEKAN,**
Prof. dr Saša Mujović






Broj: 02/1-176
Datum: 17.02.2021

Na osnovu člana 64 Statuta Univerziteta Crne Gore, u vezi sa članom 41 Pravila doktorskih studija, na predlog Komisije za doktorske studije, Vijeće Elektrotehničkog fakulteta u Podgorici, na sjednici od 11.02.2021. godine, donijelo je

ODLUKU

I Utvrđuje se da su ispunjeni uslovi iz Pravila doktorskih studija za dalji rad na doktorskoj disertaciji „**Analiza, implementacija i primjena gradijentnih algoritama za rekonstrukciju kompresivno odabranih signala**“, kandidata MSc **Stefana Vujovića**.

II Predlaže se Komisija za ocjenu navedene doktorske disertacije, u sastavu:

1. Prof. dr Irena Orović, Elektrotehnički fakultet Univerziteta Crne Gore,
2. Prof. dr Miloš Daković, Elektrotehnički fakultet Univerziteta Crne Gore,
3. Doc. dr Jonatan Lerga, Tehnički fakultet Sveučilišta u Rijeci.

Komisija iz tačke II ove Odluke podniće Izvještaj Vijeću Fakulteta u roku od 45 dana od dana imenovanja.

-VIJEĆE ELEKTROTEHNIČKOG FAKULTETA-



DEKAN, -9
N. Popović-Bugarin
Prof. dr Saša Mujović

Dostavljeno:

- Senatu,
- Centru za doktorske studije,
- u dosije,
- a/a.



Spisak radova sa rezultatima iz doktorske teze – MSc Stefan Vujović

Vodeći naučni časopisi (SCI/SCIE lista):

1. **S. Vujović**, A. Draganić, M. Lakičević, I. Orović, M. Daković, M. Beko, and S. Stanković, "Sparse Analyzer Tool for Biomedical Signals," *Sensors*, 20(9), 2602, doi: 10.3390/s20092602
2. LJ. Stanković, M. Daković, I. Stanković, and **S. Vujović**, "On the Errors in Randomly Sampled Nonsparse Signals Reconstructed with a Sparsity Assumption," *IEEE Geoscience and Remote Sensing Letters*, Vol: 14, Issue: 12, Dec. 2017, pp. 2453 - 2456, DOI: 10.1109/LGRS.2017.2768664
3. LJ. Stanković, M. Daković, and **S. Vujović**, "Reconstruction of Sparse Signals in Impulsive Disturbance Environments," *Circuits, Systems and Signal Processing*, vol. 2016, pp. 1-28, DOI: 10.1007/s00034-016-0334-3, ISSN: 0278-081X print, 1531-5878 online (online published version available on <https://rdcu.be/4XQ8>)
4. LJ. Stanković, M. Daković, and **S. Vujović**, "Adaptive Variable Step Algorithm for Missing Samples Recovery in Sparse Signals," *IET Signal Processing*, vol. 8, no. 3, pp. 246 -256, 2014. (arXiv:1309.5749v1)

Međunarodne konferencije:

1. M. Brajović, **S. Vujović**, I. Orović, and S. Stanković, "Coefficient Tresholding in the Gradient Reconstruction Algorithm for Signals Sparse in the Hermite Transform Basis," *Applications of Intelligent Systems 2018 (APPIS 2018)*, Las Palmas De Gran Canaria, 8-12 January 2018
2. S. Stanković, **S. Vujović**, I. Orović, M. Daković, and LJ. Stanković, "Combination of Gradient Based and Single Iteration Reconstruction Algorithms for Sparse Signals," *17th IEEE International Conference on Smart Technologies, IEEE EUROCON 2017*
3. **S. Vujović**, I. Stanković, M. Daković, and LJ. Stanković, "Comparison of a Gradient-Based and LASSO (ISTA) Algorithm for Sparse Signal Reconstruction," *5th Mediterranean Conference on Embedded Computing MECO 2016*, Bar, June 2016
4. **S. Vujović**, M. Daković, I. Orović, and S. Stanković, "An Architecture for Hardware Realization of Compressive Sensing Gradient Algorithm," *4th Mediterranean Conference on Embedded Computing, MECO – 2015*
5. **S. Vujović**, M. Daković, and LJ. Stanković, "Comparison of the L1-magic and the Gradient Algorithm for Sparse Signals Reconstruction," *22nd Telecommunications Forum, TELFOR, 2014*,
6. LJ. Stanković, M. Daković, and **S. Vujović**, "Concentration measures with an adaptive algorithm for processing sparse signals," *ISPA 2013*, Trieste, Italy, 4-6 September 2013, pp. 418-423



Article

Sparse Analyzer Tool for Biomedical Signals

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Abstract: The virtual (software) instrument with a statistical analyzer for testing algorithms for biomedical signals' recovery in compressive sensing (CS) scenario is presented. Various CS reconstruction algorithms are implemented with the aim to be applicable for different types of biomedical signals and different applications with under-sampled data. Incomplete sampling/sensing can be considered as a sort of signal damage, where missing data can occur as a result of noise or the incomplete signal acquisition procedure. Many approaches for recovering the missing signal parts have been developed, depending on the signal nature. Here, several approaches and their applications are presented for medical signals and images. The possibility to analyze results using different statistical parameters is provided, with the aim to choose the most suitable approach for a specific application. The instrument provides manifold possibilities such as fitting different parameters for the considered signal and testing the efficiency under different percentages of missing data. The reconstruction accuracy is measured by the mean square error (MSE) between original and reconstructed signal. Computational time is important from the aspect of power requirements, thus enabling the selection of a suitable algorithm. The instrument contains its own signal database, but there is also the possibility to load any external data for analysis.

Keywords: biomedical signals; compressive sensing; concentration measure; gradient algorithm; OMP; SIRA; statistical analyzer; sparse signal processing; TV minimization; virtual instrument

1. Introduction

The processing of under-sampled signals has attracted significant research interest in the last decade [1–10]. The signal is under-sampled if the number of available samples is less than the number of samples required by the Shannon Nyquist sampling theorem. The under-sampling can be done intentionally during the acquisition procedure or signal samples could be lost during the transmission or discarded owing to noise. Intentional omitting of signal samples and their later reconstruction found its usage in the applications dealing with a large number of signal samples, with the purpose of increasing the processing speed. Such an approach could be important in medical applications. For example, in magnetic resonance imaging (MRI), lowering the required amount of data reduces the time of patient exposure to the harmful MR waves [11].

With the increasing use of wireless technology and smart devices in our everyday life, portable medical devices become very popular [12–14]. Modern technology is changing the way medicine approaches various health problems in a number of diseases and for a large number of patients. Using portable medical devices, the possibility to monitor patient condition every moment and in every place

is provided. Technology can ease life to many patients, especially those with chronic diseases, patients with diabetes, those with cardiovascular diseases, or elderly people [13]. For example, automatic reminders for taking medications are developed, as well as devices for monitoring different health parameters [14,15] such as body temperature, glucose levels, blood pressure, and so on. Real-time monitoring and on-time reactions can save the patient's life in situations when there is no need for hospitalization, but the vital parameters should constantly be tracked. Monitoring in such a way improves patient comfort, while at the same time unloading the hospital capacities in terms of staff and space [13].

Another important aspect of digital monitoring is the opportunity to store collected data in the patient's medical record, from where it can be easily transferred to hospitals and clinicians all over the world. The diagnosis could be provided at a distance, without having physical contact with the patient. This has numerous advantages: avoiding going to the hospitals and waiting, getting opinions from many professionals around the world, and so on.

The communication between the patient and the healthcare professional should be fast and secure. The intentional under-sampling of medical data could lead to dealing with a much smaller amount of data, and thus faster transmission [11]. Moreover, sending only part of the original signal keeps the information sent secure. At the receiver part, the under-sampled signal should be recovered and back to its original version. The special problem during the transmission could be noise. Noisy samples could be considered as missing and, if they are detected, lead to signal under-sampling [4].

One of the new areas that enables under-sampled signal recovery is called compressive sensing (CS). The idea behind the concept of CS is to reduce the sampling rate far below that defined by the Nyquist–Shannon sampling theorem, and later recover the whole signal by applying complex mathematical algorithms [2,4,11,16–32]. Therefore, the CS represents a completely new paradigm compared with the traditional sensing strategy, and has been used in various applications such as image processing, biomedical signals, wireless communications, and radars [26–30]. In the CS scenario, the signal can be completely recovered from a small set of randomly acquired linear measurements, if the signal of interest has a sparse representation when represented in a certain transformation basis. Sparse representation means that the signal can be represented by a few non-zero coefficients, which is much lower than its original dimension. Consequently, for signals in real-world applications, it is important to identify sparse representations using the appropriate dictionaries of atoms or transformations.

Here, the application of this attractive approach to the various signals in biomedicine will be presented, providing clinicians and researchers with a good base for possible improvement of different technology-based services and devices used in medicine. The proposed instrument implements a number of algorithms in order to perform reconstruction, as well as to compare the results obtained with different algorithms. This tool can be used to create a number of different signals, with a specially designed panel where all important signal parameters can be easily chosen, such as signal sparsity, percent of missing samples, signal length, and so on. Finally, the paper combines several algorithms that solve the reconstruction problem using quite different mathematical approaches, also contributing to the educational dimension of the proposed instrument. The software provides the possibility to compare several different algorithms in terms of reconstruction precision (expressed in terms of mean square error (MSE)) and achieved sparsity level (expressed in terms of concentration measures ℓ_1 -norm and Gini index). On the basis of the comparison results, the user chooses the transform and the algorithm that best suit the considered signal type. Moreover, the user can choose the solution that provides the best sparsity, or the lowest reconstruction error, or as a third option, a solution providing the best trade-off between these two measures.

By providing the analytical visualization of results, together with reconstruction error and concentration measures, the tool allows users to test and choose the best possible optimization approach and the most suitable sparse domain for the applications with biomedical signals or images. Namely, among a number of algorithms for sparse signal recovery and transform domain

modeling, the proposed tool allows the selection of an appropriate approach combined with the suitable transformation for achieving the most accurate reconstruction results for the considered signal types. The tool is designed in a user-friendly manner even for the non-specialists in the field, as it does not require certain pre-knowledge about the implemented approaches. On the basis of the statistical parameters for measuring reconstruction efficiency, the users are able to choose the most suitable among the offered solution. However, it is also convenient for the researchers working in the field, as it provides a set of comprehensive solutions for the processing of biomedical data that can be further extended or adapted for different purposes.

The paper is structured as follows. Section 2 provides a theoretical background on the CS. Section 3 describes approaches for signal recovery that are implemented within the instrument. Section 4 describes the software, while Section 5 gives the results applied to concrete biomedical signals and images. The conclusion is given in Section 6.

2. Theoretical Background

Consider a time-domain signal $x(n)$, composed of N samples, that is, $n = 0, 1, \dots, N - 1$. Suppose that the arbitrary linear transform of this signal is denoted with $X(k)$, where $k = 0, 1, \dots, N - 1$.

If the most of the coefficients of $X(k)$ are zero-valued or negligible, then $X(k)$ represents a sparse presentation of signal $x(n)$. For a K -sparse signal, it might be said that only K coefficients of signal $X(k)$ are non-zero, where $K \ll N$.

Signal $x(n)$ and $X(k)$ are related via the following [2,4]:

$$X(k) = \sum_{n=0}^{N-1} x(n)\psi_k^*(n), \quad x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\psi_k(n), \quad (1)$$

where $\psi_k(n)$ and its inverse $\psi_k^*(n)$ are basis functions. In the case of the discrete fourier transform (DFT) as one of the commonly used transforms, the basis function equals $\psi_k(n) = e^{j2\pi nk/N}$. Note that, in general, CS algorithms deal with arbitrary linear transformations like discrete cosine transform (DCT), Hermitian transform (HT), wavelet transform [33,34], and so on, while some of the algorithms like the gradient-based one [11,32] deal even with nonlinear transformations. In this work, the HT, DCT, and DFT transforms are considered, as they found many applications in biomedical data representation and analysis [35–39]. For example, DFT is used for the analysis of electroencephalography (EEG) signals and blood pressure signals [35], and DCT and DFT transforms are used in MRI image processing [36,37], while the HT is used in electrocardiograph (ECG) and QRS signal analysis [38,39].

Discrete Hermite function of an order p is defined as follows [5,6]:

$$\psi_p(t) = \frac{e^{-\frac{t^2}{2\lambda^2}} H_p(t/\lambda)}{\sqrt{\lambda 2^p p! \sqrt{\pi}}}, \quad H_p(t) = (-1)^p e^{t^2} \frac{d^p (e^{-t^2})}{dt^p}, \quad (2)$$

where H_p is a Hermite polynomial of the p -th order, λ is the scaling factor used for “stretching” or “compressing” the Hermite functions, while the HT matrix can be defined as follows:

$$\Psi = \frac{1}{M} \begin{bmatrix} \frac{\psi_0(n_1)}{(\psi_{M-1}(n_1))^2} & \vdots & \frac{\psi_0(n_M)}{(\psi_{M-1}(n_M))^2} \\ \frac{\psi_1(n_1)}{(\psi_{M-1}(n_1))^2} & \vdots & \frac{\psi_1(n_M)}{(\psi_{M-1}(n_M))^2} \\ \dots & \vdots & \dots \\ \frac{\psi_{M-1}(n_1)}{(\psi_{M-1}(n_1))^2} & \vdots & \frac{\psi_{M-1}(n_M)}{(\psi_{M-1}(n_M))^2} \end{bmatrix}. \quad (3)$$

The “stretching” or “compressing” of the Hermite functions is used for better fitting of the function to the signal [38], providing better sparsity of the signal in the HT domain.

When dealing with the HT, signal $x(n)$ should be sampled at the non-uniform points that correspond to the roots of the Hermite polynomial. Another approach is to interpolate the uniformly sampled signal in order to obtain requested signal values at the non-uniform points. Hermite functions obtained by the uniform sampling of the corresponding continuous functions are not orthogonal. Therefore, in order to obtain the orthogonal function, the interpolation of the Hermite function is used. The second approach, interpolation, is also implemented within the instrument. The signal sampled according to the sampling theorem is interpolated using the sinc interpolation formula [11,27]:

$$x(\sigma n_m) \approx \sum_{i=-K}^K x(iT) \frac{\sin(\pi(\sigma n_m - iT)/T)}{\pi(\sigma n_m - iT)/T}, \quad m = 1, \dots, M. \quad (4)$$

where T denotes the sampling period. The optimal value of the parameter σ produces the best possible concentration in the transform domain, and it can be found using the ℓ_1 -norm optimization:

$$\sigma_{opt} = \min_{\sigma} \|HT\{x(\sigma n_m)\}\|_1. \quad (5)$$

The instrument implements several commonly used sparsity measures that are suitable for the observed signals. Besides the ℓ_1 -norm concentration, there are a lot of approaches for measuring sparsity, for example, entropy based approaches [40,41] or the Gini index [41–44]. The Gini index satisfies most of the desirable characteristics of measures of sparsity and overcomes the limitations of standard norm-based sparsity measures, as proven in [41]. It is suitable for comparing the sparsity of a signal in different transform domains [42], and is also used as a measure of sparsity for biomedical signals [43,44]; therefore, this measure is implemented within the instrument together with the ℓ_1 -norm concentration. The Gini index is calculated according to the following relation:

$$G(x) = 1 - 2 \sum_{i=1}^N \frac{|x_s(i)|}{\|x\|_1} \left(\frac{N-i+\frac{1}{2}}{N} \right), \quad (6)$$

where x_s is sorted version of a set of elements in ascending order. The Gini index values can be between 0 and 1. A higher value of the Gini index corresponds to better sparsity.

Here, the HT transform is mainly implemented for the application in electrocardiograph (ECG) signals and their QRS complexes. An ECG signal represents the electrical activity of a heart over time, while the QRS complex is formed by three of the graphic deflections of the ECG signal [45]. The analysis of the ECG signals and QRS complexes is used in heart function monitoring and disease diagnostic. The interval between two successive QRS provides information about the regularity of the cardiac rhythm. Moreover, the QRS observation is used in the diagnosis of other heart abnormalities such as myocardial infarction or arrhythmia and, therefore, the QRS represents an important part of the ECG signal. Having in mind that the Hermite functions show physical similarity with QRS complexes, they are found to be a suitable tool for their analysis [6,38,46].

Another observed transformation, DCT, has the following form:

$$DCT(k) = c(k) \sum_{n=0}^{N-1} x(n) \cos \frac{(2n+1)k\pi}{2N}, \quad (7)$$

where $k = 0, \dots, N-1$ and $c(k)$ is as follows:

$$c(k) = \begin{cases} \sqrt{1/N}, & k = 0 \\ \sqrt{2/N}, & k = 1, \dots, N-1 \end{cases}. \quad (8)$$

In the matrix notation, the above equations can be generally written as follows: $\mathbf{X} = \mathbf{\Psi}\mathbf{x}$ and $\mathbf{x} = \mathbf{\Psi}^{-1}\mathbf{X}$, where the vector \mathbf{X} has elements $X(k)$, and vector \mathbf{x} has elements $x(n)$. Both vectors are of length N , and $\mathbf{\Psi}$ is $N \times N$ transform matrix with elements $\psi_k^*(n)$.

Suppose an M -length vector \mathbf{y} , which is a linear combination of elements from vector \mathbf{X} . This vector is obtained as follows [4]:

$$\mathbf{y} = \mathbf{A}\mathbf{X}, \quad (9)$$

where \mathbf{A} is an $M \times N$ matrix. The CS task is to reconstruct signal \mathbf{X} (or \mathbf{x}) from vector \mathbf{y} . Note that the length of vector \mathbf{y} is lower than the length of \mathbf{X} , because ($M < N$). Construction of matrix \mathbf{A} attracts significant research interest. Namely, the randomly constructed sensing matrix is considered in all implemented algorithms. The sensing matrix is a random matrix $\mathbf{\Phi}$ that is multiplied by the transform matrix $\mathbf{\Psi}$, resulting in the compressive sensing matrix \mathbf{A} . The compressive sensing matrix \mathbf{A} is called the random partial transform domain matrix containing the random combination of rows from $\mathbf{\Psi}$ that corresponds to the random position of the available samples. The vector \mathbf{y} of length M is equal to samples of signal \mathbf{x} , taken at the positions corresponding to preserved rows in matrix $\mathbf{\Psi}^{-1}$, that is,

$$y(i) = x(n_i), \quad i = 1, 2, \dots, M. \quad (10)$$

The reconstruction error may depend on the choice of the domain of sparsity and reconstruction algorithm, but in certain cases, it may also be the consequence of the quantization influence, as discussed in [47]. Namely, the limitations of the number of bits used for the representation of the available signal samples can affect the reconstruction performance. If the measurements $y(m)$ are normalized to the range $-1 \leq y(m) \leq 1$, and the B -bit registers are assumed, then sparse coefficients $X(i)$ have to be within the range $-\min\{\sqrt{M/K}, 1\} < X(i) < \min\{\sqrt{M/K}, 1\}$, in order for product $\mathbf{y} = \mathbf{A}\mathbf{X}$ to produce amplitudes below 1 [47]. The reconstruction error related to the number of bits is given by [47] $e^2 = 3.01 \times \log_2 K - 6.02B - 7.78$. The proposed solution is designed for the 64-bit computer device, so the effects of quantization in this case can be considered negligible. However, the quantization issues should be carefully considered especially for hardware realizations of considered reconstruction algorithms.

3. Approaches for Under-Sampled Signal Reconstruction

Another direction within the CS area is related to the reconstruction algorithms for compressed sensed data. Many signal reconstruction algorithms have been proposed depending on the type of signal and CS scenario [4,11,16–32]. The performance of these algorithms may vary depending on the number of missing samples, level of sparsity, and amount of external noise, and there is still a lack of general instructions for their practical use.

Among the algorithms for 1D signals' reconstruction that are included in the Virtual instrument, the ℓ_1 -magic algorithm is included from the class of convex algorithms. Next, the orthogonal matching pursuit (OMP) algorithm [21] from the class of greedy algorithms is implemented, as well as the single iteration construction algorithm (SIRA), based on the analytical threshold derivations [9,11,26] and generalized deviation-based reconstruction algorithm [23]. Finally, as an efficient and simple solution, the gradient-based convex algorithm is implemented [32]. This algorithm suits for a general class of signals, and for both 1D and 2D cases. It can be successfully used when the measurements are affected by the noise and also provides satisfactory results for the natural images reconstruction from a very reduced set of pixels. Greedy approaches such as SIRA, OMP, and generalized deviation-based reconstruction algorithm (GDBRA) are faster, but less precise compared with convex optimization algorithms and also require a priori knowledge about the signal (e.g., a number of components). The implemented reconstruction algorithms are briefly described in the sequel.

3.1. $L1$ -Magic

The solution of problem (9) requires exhaustive searches over subsets of columns of the CS matrix \mathbf{A} and, therefore, is not computationally feasible. Computationally more suitable approaches

solve a convex optimization problem through linear programming, such as basis pursuit (BP), basis pursuit de-noising (BPDN), least angle regression (LARS), least absolute shrinkage and selection operator (LASSO) [4,11,18,21,22], and so on. The near-optimal approach is provided using the convex ℓ_1 -minimization. It is defined as follows:

$$\min\|\mathbf{X}\|_1 \text{ subject to } \mathbf{y} = \mathbf{A}\mathbf{X}. \quad (11)$$

Standard linear program form can be recast as follows [22]:

$$\min_{\mathbf{X}} \langle \mathbf{c}_0, \mathbf{X} \rangle \text{ subject to } \mathbf{y} = \mathbf{A}\mathbf{X}, f_i(\mathbf{X}) \leq 0. \quad (12)$$

Each of $f_i(\mathbf{X}) = \langle \mathbf{c}_i, \mathbf{X} \rangle + d_i$, $i = 1, \dots, m$ is a linear functional for $\mathbf{c}_i \in R^N$, $d_i \in R$. At the optimal point, there will exist dual vectors $\mathbf{v}^* \in R^K$ and $\lambda^* \in R^M$ that satisfy Karush–Kuhn–Tucker conditions:

$$\begin{aligned} \mathbf{c}_0 + \mathbf{A}^T \mathbf{v}^* + \sum_i \lambda_i^* \mathbf{c}_i &= 0, \quad \lambda_i^* f_i(\tilde{\mathbf{X}}) = 0, \quad i = 1, \dots, m \\ \mathbf{A}\tilde{\mathbf{X}} &= \mathbf{y}, \quad f_i(\tilde{\mathbf{X}}) \leq 0, \quad i = 1, \dots, m. \end{aligned} \quad (13)$$

The algorithm finds the needed vector (optimal dual vector) by solving the system of nonlinear equations. At the interior point $(\tilde{\mathbf{X}}, \mathbf{v}^*, \lambda^*)$, the system is linearized and solved.

3.2. Gradient-Based Algorithm

The recently proposed gradient algorithm [32] for sparse signal reconstruction is a computationally efficient algorithm that belongs to a wide class of gradient CS algorithms. The idea behind this algorithm is to observe missing samples in a dense domain as the variables, which are reconstructed in a way to produce minimal concentration measure in the sparse domain. Reconstruction of missing samples is the main difference of this algorithm compared with the others, which mainly reconstruct the signal in the sparse domain. The implementation steps for the 1D signal are given in Algorithm 1.

3.3. SIRA—Single Iteration Reconstruction Algorithm

Single iteration reconstruction algorithm [3,6,7,25] is based on the idea of separating signal components in the sparsity domain from the noise components that appear as the consequence of missing samples. The required assumption is that all signal components are above the calculated threshold, while the noise components are under the threshold. The probability that values corresponding to noise are lower than T is $P(T)$.

Depending on the sparsity domain, the threshold is calculated according to the relations derived in [6,7,25]. In the case of the DFT as a sparsity domain, the threshold T_{DFT} is as follows:

$$T_{DFT} = \sqrt{-\sigma_{MS}^2 \log(1 - P(T) \frac{1}{N^2})} \approx \sqrt{-\sigma_{MS}^2 \log(1 - P(T) \frac{1}{N})}, \quad (14)$$

If the DCT is considered as a transformation domain, the threshold T_{DCT} is calculated as follows:

$$T_{DCT} = 4 \sqrt{\frac{M(N-M)}{N^2(N-1)}} K, \quad (15)$$

while in the HT domain, the threshold T_{HT} is follows:

$$T_{HT} = \sigma_{MS} \sqrt{\left(-4/\pi - aL + \sqrt{(4/\pi + aL)^2 - 4aL} \right) / a}, \quad (16)$$

where $a \leftarrow 0.147$, $L \leftarrow \log(1 - (P(T))^{2/M})$ and $P(T)$ is desired probability (e.g., 0.99).

Algorithm 1. Gradient-based algorithm

Input: set of the positions of the missing samples: \mathbb{N}_x ; measurement vector y ;

Set $x^{(0)}(n) \leftarrow y(n)$ for $n \notin \mathbb{N}_x$

Set $x^{(0)}(n) \leftarrow 0$ for $n \in \mathbb{N}_x$

$m \leftarrow 0$

Set $\Delta \leftarrow \max |x^{(0)}(n)|$

repeat

repeat

$x^{(m+1)}(n) \leftarrow x^{(m)}(n)$, for each n

for $n_i \in \mathbb{N}_x$ **do**

$X^+(k) \leftarrow \mathfrak{I}\{x^{(m)}(n) + \Delta\delta(n - n_i)\}$,

$X^-(k) \leftarrow \mathfrak{I}\{x^{(m)}(n) - \Delta\delta(n - n_i)\}$, (\mathfrak{I} - transformation)

$g(n_i) \leftarrow \frac{1}{N} \sum_{k=0}^{N-1} (\|X^+(k)\|_1 - \|X^-(k)\|_1)$,

$x^{(m+1)}(n_i) \leftarrow x^{(m)}(n_i) - g(n_i)$

end for

$m \leftarrow m + 1$

until stopping criterion is satisfied

$\Delta \leftarrow \Delta/3$

until required precision is achieved

Output: reconstructed signal $x^{(m)}(n)$

In Equations (14)–(16), M denotes the number of missing samples, N is the signal length, and K is the sparsity. The parameter $N - K$ in T_{DFT} could be approximated as N , based on the fact that $K \ll N$. The steps of the algorithm are given in Algorithm 2. The resulting vector \mathbf{X} contains the signal components values X_R at positions \mathbf{k} , while the rest of the transform domain coefficients are zero.

Algorithm 2. SIRA

Input:

Measurement vector y ; $M \times N$ matrix \mathbf{A} ; signal sparsity K

- Set desired probability (e.g., $P \leftarrow 0.99$)
- Calculate variance—variance is calculated by using one of the relations, depending on the chosen transformation domain. The corresponding equations are in the following table:

Transformation domain	DFT	DCT	HT
Variance	$\sigma_{MS}^2 = M \frac{N-M}{N-1} \sum_{i=1}^M \frac{y(i)^2}{M}$	$\sigma_{MS}^2 = M \frac{N-M}{N^2(N-1)} \sum_{i=1}^K A_i^2$	$\sigma_{MS}^2 = \frac{(N-M)M-(N-M)^2}{M^2(M-1)} \sum_{i=1}^K A_i^2$
		$\sum_{i=1}^K A_i^2 = \frac{N}{M} \sum_{n \in M} s^2(n)$	
		$s(n)$ – available samples	

- For a given P calculate threshold according to relation (13);
- Calculate the initial transform domain vector \mathbf{X}_0 : $\mathbf{X}_0 = \mathbf{A}^{-1}y$;
- Find positions of components in \mathbf{X} higher than normalized threshold T , $\mathbf{k} = \arg\{|\mathbf{X}_0| > T/N\}$;
- Form CS matrix by using only \mathbf{k} columns from \mathbf{A} , $\mathbf{A}_{CS} \leftarrow \mathbf{A}(\mathbf{k})$
- Calculate $X_R = (\mathbf{A}_{CS}^T \mathbf{A}_{CS})^{-1} \mathbf{A}_{CS}^T y$;

Output: X_R

3.4. GDBRA—Generalized Deviation-Based Reconstruction Algorithm

This algorithm uses the model of general deviations in the transform domain instead of transform representation itself. The algorithm works with signals sparse in the DFT domain, and the generalized deviations are derived for the DFT case. The use of generalized deviations is inspired by the robust statistics theory, where the form of transformation is adapted to the specific noise environment. Consequently, this approach provides flexibility of using different types of minimization norms for different noisy environments. The algorithm [23] is described through the steps in Algorithm 3.

Algorithm 3. GDBRA

- **Step 1:** For each $k = 0, 1, \dots, N - 1$ calculate generalized deviation $GD(k)$ as:

$$GD(k) = \underset{n_m \in N_{avail}}{\text{mean}} \left\{ \left| x(n_m) e^{-j2\pi k n_m / N} - \underset{n_m \in N_{avail}}{\text{mean}} \left\{ x(n_1) e^{-j2\pi \frac{k n_1}{N}}, \dots, x(n_M) e^{-j2\pi \frac{k n_M}{N}} \right\} \right|^L \right\},$$

where $\underset{n_m \in N_{avail}}{\text{mean}} [f(n_M)]$ is used to find mean value of vector with elements $f(n_M)$, for $n_M \in N_{avail}$.

For norm ℓ_2 use $L = 2$, while for norm ℓ_1 use $L = 1$.

- **Step 2:** Determine the signal support $\mathbf{k} = \arg\{GD(k) < T\}$ where T can be calculated with respect to $\text{median}\{GD(k)\}$, or $p \cdot \text{median}\{GD(k)\}$ (where p is constant close to 1) for example, but also with respect to mean or minimal value. The vector of positions \mathbf{k} should contain all signal frequencies $k_i \in \mathbf{k}$ for any $i = 1, \dots, K$.
- **Step 3:** Set $\tilde{X}(k) = 0$ for frequencies $k \notin \mathbf{k}$;
- **Step 4:** the estimates of the DFT values can be calculated by solving the set of equations, at the localized frequencies from the vector \mathbf{k} , where \mathbf{k} contains K signal frequencies $\mathbf{k} = k_1, k_2, \dots, k_K$. A system of equations is set as follows:

$$\sum_{i=1}^K X(k_i) e^{j2\pi k_i n_m} = x(n_m)$$

- **Step 5:** the CS matrix A_{CS} is formed as a partial DFT matrix: columns correspond to the positions of available samples, rows correspond to the selected frequencies. The system is solved in the least square sense:

$$\mathbf{X} = (A_{CS}^T A_{CS})^{-1} A_{CS}^T \mathbf{y}.$$

3.5. OMP—Orthogonal Matching Pursuit

Orthogonal matching pursuit (OMP) is a kind of greedy algorithm that finds the best correlation between measurements vector \mathbf{y} and the matrix \mathbf{A} through iterations. In each iteration, the column of \mathbf{A} corresponding to certain sparse domain coefficient is found. The OMP implementation is given in Algorithm 4.

3.6. TV Minimization

Another approach, the total variation (TV) minimization, is implemented in the Virtual instrument. This approach can be successfully applied to both 1D and 2D signals. Generally, the images are not strictly sparse either in the spatial or in the space domain. Therefore, the ℓ_1 -minimization of the image gradient, named TV minimization, is found to be a more suitable approach than minimizing the image coefficients. This approach is suitable for noisy signals as well and is described with the following relation:

$$\|\mathbf{x}\|_{TV} = \sum_{i,j} |(\nabla \mathbf{x})_{ij}|, \text{ where } \nabla_{i,j} \mathbf{x} = \begin{bmatrix} x(i+1, j) - x(i, j) \\ x(i, j+1) - x(i, j) \end{bmatrix} \quad (17)$$

In this paper, TV minimization is combined with the DFT and DCT transformations.

Algorithm 4: Orthogonal matching pursuit

Input: Measurement vector y , $M \times N$ matrix A , signal sparsity K

- Set initial residual: $r_0 \leftarrow y$
- Set initial indices: $\Omega_0 \leftarrow \emptyset$
- Set matrix of chosen columns: $\Theta_0 \leftarrow []$
- for $i \leftarrow 1$ to K
 - $\omega_i = \underset{j=1,\dots,N}{\operatorname{argmax}} |\langle r_{i-1}, A_j \rangle|$ - maximum correlation column
 - $\Omega_i \leftarrow \Omega_{i-1} \cup \omega_i$ - update set of indices
 - $\Theta_i \leftarrow [\Theta_{i-1} A_{\omega_i}]$ -
 - update set of chosen columns $x_i = \underset{x}{\operatorname{argmin}} \|r_{i-1} - \Theta_i x\|_2^2$ $a_i \leftarrow \Theta_i x_i$
 - $r_i \leftarrow y - a_i$
- end for

Output: x_K and Ω_K

3.7. Douglas–Rachford Algorithm

An efficient solution for 2D signal recovery is based on the Douglas–Rachford (DR) algorithm and its special case, alternating directions methods of multipliers (ADMM) [31,48]. The ADMM, that is, a variation on the method of multipliers, is a special case of Douglas–Rachford splitting. The DR problem is defined as follows [31–48]:

$$\underset{x \in \mathbb{R}^N}{\operatorname{minimize}} (f(x) + g(x)), \quad (18)$$

where f and g are convex functions that should not be smooth, but only proximable. The proximal mappings for f and g are computed as follows [48]:

$$\operatorname{prox}_{\lambda f}(x) = \underset{y}{\operatorname{argmin}} \frac{1}{2} \|x - y\|^2 + \lambda f(y), \quad \operatorname{prox}_{\lambda g}(x) = \underset{y}{\operatorname{argmin}} \frac{1}{2} \|x - y\|^2 + \lambda g(y). \quad (19)$$

In CS terms, $f(x) = \eta_H$ and $g(x) = \|x\|_1$, where the affine space H is defined as $H = \{x: Ax = y\}$, while η_H is an indicator function [48]:

$$\eta_H(x) = \begin{cases} 0 & \text{if } x \in H, \\ +\infty & \text{if } x \notin H. \end{cases} \quad (20)$$

Therefore, the proximal operators for functions $f(x)$ and $g(x)$ are as follows:

$$\begin{aligned} \operatorname{prox}_g(x) &= \max\left(0, 1 - \frac{\lambda}{|x|}\right)x, \\ \operatorname{prox}_f(x) &= \operatorname{prox}_{\lambda \eta_H}(x) = x + A^T (AA^T)^{-1} (y - Ax). \end{aligned} \quad (21)$$

The DR iteratively finds a solution according to the following relation:

$$\tilde{x}_{k+1} = (1 - \mu/2)\tilde{x}_k + \frac{\mu}{2} r \operatorname{prox}_{\lambda g}(r \operatorname{prox}_{\lambda f}(\tilde{x}_k)), \quad 0 < \mu < 2, \quad (22)$$

where $r \operatorname{prox}$ denotes reversed-proximal mappings given (for function h):

$$r \operatorname{prox}_{\lambda h} = 2 \operatorname{prox}_{\lambda h} - h(x). \quad (23)$$

4. Design of the Software—Virtual Instrument for Biomedical Signals Reconstruction

The detailed description of the developed software is described in the sequel. The proposed instrument is designed to work with various types of biomedical signals (both the 1D and 2D signals).

The instrument is implemented in MATLAB 2017a version. The flowchart of the instrument is shown in Figure 1.

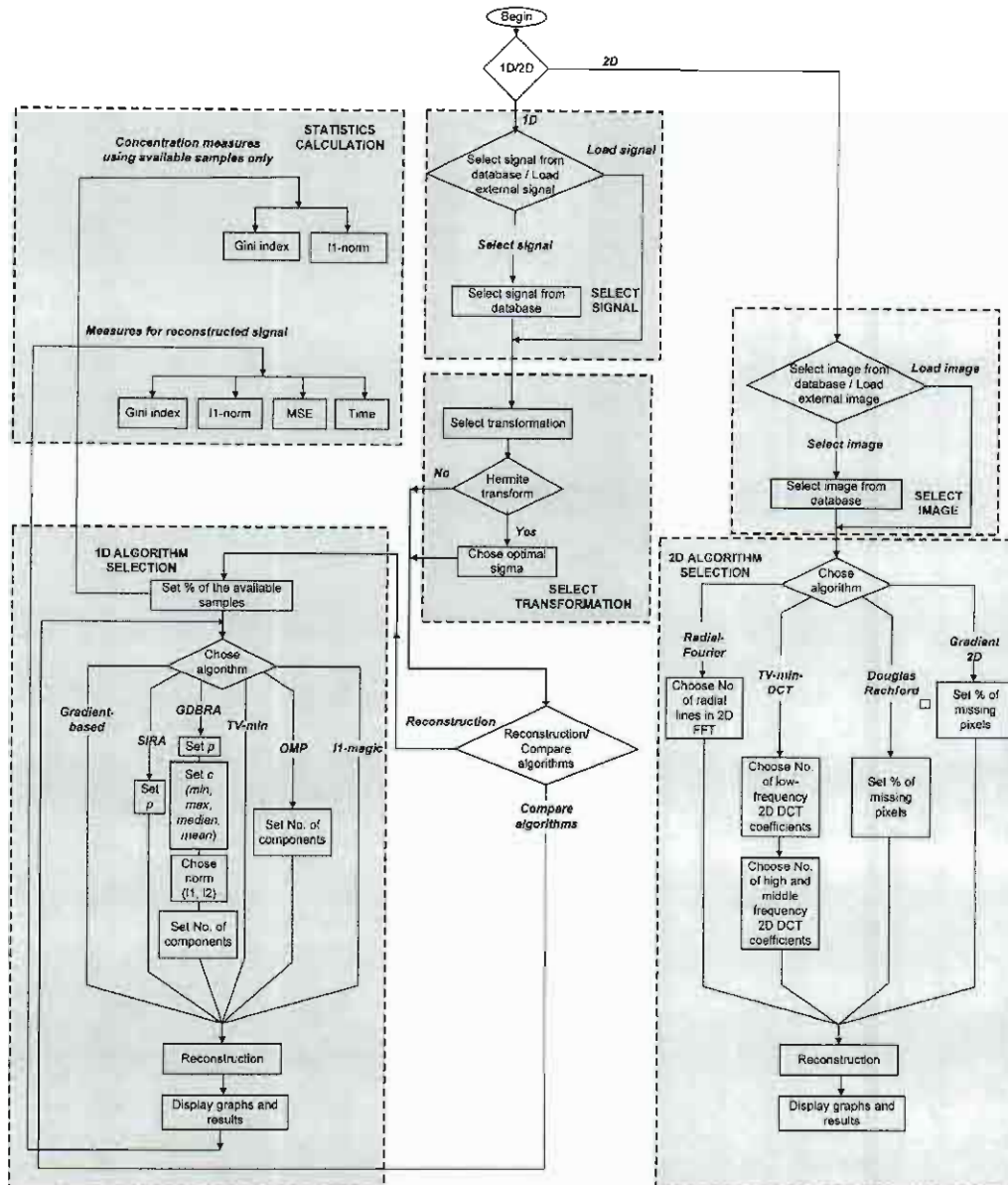


Figure 1. The flowchart for the virtual instrument realization. MSE, mean square error; SIRA, single iteration construction algorithm; OMP, orthogonal matching pursuit; GDBRA, generalized deviation-based reconstruction algorithm; DCT, discrete cosine transform; FFT, fast Fourier transform.

There is a specially designed panel where the required parameters for signal reconstruction can be easily chosen, for instance, the percent of missing samples, sparse transformation domain, and so on. The following reconstruction algorithms are implemented: ℓ_1 -magic algorithm and gradient-based algorithm from the convex optimization group, as well as the OMP algorithm, SIRA, and GDBRA from the greedy approaches group. The gradient-based approach is also suitable for 2D signals together with TV minimization, which belongs to the convex optimization group. Another 2D recovery approach is Douglas–Rachford splitting, implemented in several commonly used solvers for sparse reconstruction, such as YALL1, SALSA, and SpaRSA.

A special section of the Virtual instrument is devoted to the statistical analyzer, which provides some interesting statistical parameters for the evaluation of reconstruction algorithms, such as mean square error (MSE) and two concentration measures used for choosing the most suitable transform or the most suitable reconstruction algorithm. The second part of the proposed Virtual instrument is dedicated to 2D signals, that is, biomedical images. A number of images are provided in order to test several algorithms for sparse recovery.

4.1. Part 1—Reconstruction of 1D Signals

The outlook of the solution for 1D signal recovery is presented in Figure 2. The instrument includes all described reconstruction algorithms and their relevant parameters, with the possibility for further extension with additional methods. In order to provide a user-friendly environment, the instrument is structured through few sections that will be explained in detail.

Section 1—Signal parameters adjustment: This part of the instrument is used to generate different types of signals in order to test implemented algorithms. The database contains ECG signals, extracted QRS complexes (there is also the option for users to extract QRS using the instrument), signals from respiration monitor, and brain activity signals such as electroencephalography (EEG) and electrooculography (EOG) signals. Some signals are taken from the open biomedical signals databases [49,50], while the others (e.g., respiration signal) are recorded using the National Instruments Elvis platform.

The first option within the instrument is to choose a signal from the drop-down menu, or to load a certain signal by using the *Load external signal* option. The measurements are obtained from time-domain samples, so there is an option to set a percentage of available data (e.g., time-domain samples). For example, if the percent of available samples is set to 25% for the signal length of 256 samples, then the length of measurement vector y will be 64, and the remaining 192 samples have to be recovered.

Furthermore, the options for sparse transform domain selection are provided. The transform can be selected from the drop-down menu. Selecting a transformation automatically calculates the concentration measure in the statistical analyzer panel. This is useful if the user does not have prior knowledge about the signal and its sparse transform domain. Two measures are provided, the ℓ_1 -norm and the Gini index, and these are calculated using the transform domain coefficients. The transformation that provides minimal ℓ_1 -norm or maximal value of Gini index is the sparsest among those observed. However, for choosing the most effective reconstruction algorithm, the MSE between the original and reconstructed signals should also be taken into account together with the concentration measures.

In the drop-down menu, the user can choose one of the commonly used transform basis: DFT, DCT, or HT. These are shown to be suitable for most of the considered biomedical signals [5,11,46]. It is important to note that, for the QRS complexes, the most suitable transformation is HT, owing to the similarity between the QRS signals and the Hermite functions. Therefore, for QRS, there is an option to optimize HT by choosing the sigma parameter.

In this part of the Virtual instrument, there is also a drop-down menu for choosing one of the offered algorithms. On the basis of the chosen algorithm, the panel changes and additional options appear, showing algorithm input parameters. Gradient and ℓ_1 -magic algorithms do not need any additional parameters, while OMP requires an expected number of components in the sparse domain; SIRA requires a value for probability P ; while GDBRA requires four parameters, p , c , norm selection, and a number of components in the sparse domain (p is constant that has values between 0 and 1, while c can be 1 for a max, 2 for a min, 3 for a mean, or 4 for a median in generalized deviation calculation). TV minimization requires a number of components in the low and middle frequencies, respectively.

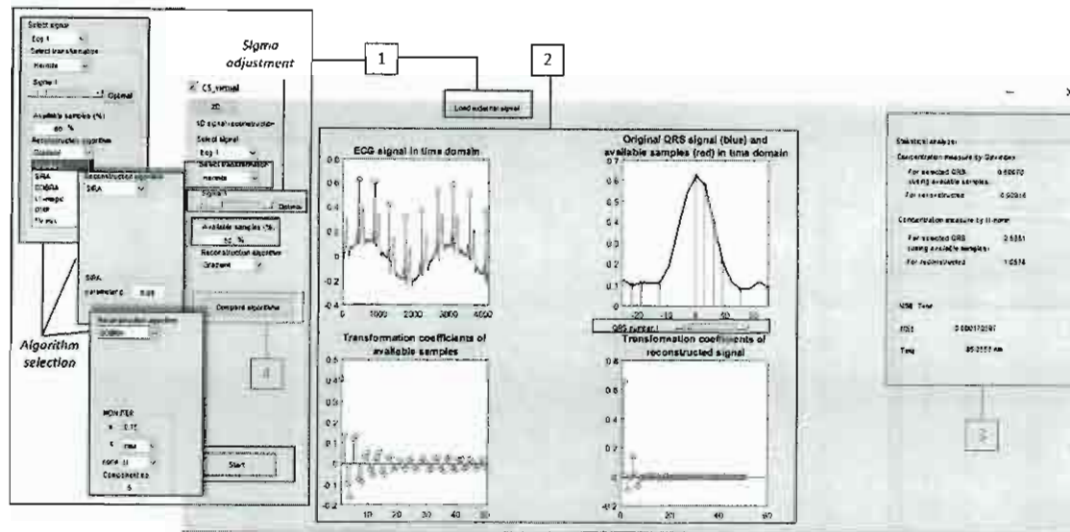


Figure 2. The outlook of part for 1D signal analysis. ECG, electrocardiograph.

The part for the choice of optimal sigma parameter in HT according to Relation (5) is provided. The sigma can be changed using the scroll button. Moreover, the optimal value, providing the best possible concentration for the selected signal, can be automatically calculated by pressing the *Optimal* button. For other transforms, the value providing the best sparsity is chosen with the help of concentration measure values—the ℓ_1 -norm and the Gini index. The reconstruction is initiated by pressing the *Start* button.

Section 2—Graphical presentation of results. This is part of the instrument where the results are graphically presented. Depending on the chosen algorithm, there are several graphics within this section, as presented in Figure 2 (block denoted by number 2). For the ECG signal, there is a possibility to test the reconstruction on the whole signal, but also to choose and observe the QRS complexes using the scroll button. Here, the original and recovered signal are displayed. Moreover, red marks on the selected QRS complex denote the available samples. The transformation can be calculated and displayed for the available samples only, as shown in the panel. The reconstructed sparse representation is presented in the special graph within this section.

Section 3—Statistical analyzer: An important part of the proposed instrument is the statistical analyzer part. Here, the concentration measures by the ℓ_1 -norm and Gini are calculated for the available samples of the signal, as well as for the signal after the reconstruction. After the signal is recovered using the chosen algorithm, the MSE between the original and reconstructed signal (calculated in the time domain) is displayed on the last part of the statistical analyzer. Concurrently, the execution time of the algorithm is also presented in the same panel. Figure 3 shows the performance of the statistical analyzer demonstrated on the example of the gradient-based reconstruction algorithm.

Example in Figure 2 shows the whole ECG signal on the upper left graph, while the selected QRS complex is shown in the upper right graph. The scroll button below the QRS graph enables QRS selection. HT is selected as a sparsity domain, while the percentage of the available samples is set to 50%. The optimal value for the sigma parameter is used, calculated according to (5). The graph of selected QRS shows also the available samples denoted by red marks. Two lower graphs show HT transform of the available samples (left lower graph) and (sparse) HT of the reconstructed signal. The statistical analyzer displays concentration measures, ℓ_1 -norm, and Gini index: (1) measures are calculated using available samples of the selected QRS and (2) using the reconstructed signal. It can be seen that ℓ_1 -norm has a lower value for reconstructed signal compared with the value when it is calculated for available samples only, while the value of the Gini index is higher for the recovered signal, as expected.

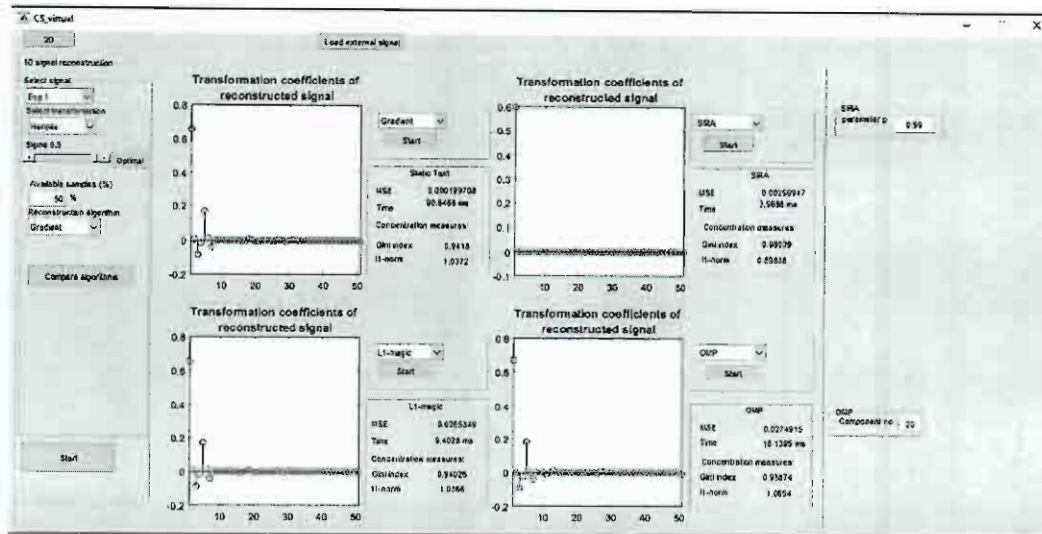


Figure 3. The outlook of the Comparison of the algorithms part applied to the QRS signal recovery.

Section 4—Compare algorithms: the virtual instrument is designed in a way to provide a comparison of the reconstruction results achieved by different algorithms. This is an important part of the instrument, as it enables testing the most suitable approach for a considered biomedical signal or image, in terms of reconstruction quality and processing speed. For this reason, an additional set up section is included and it provides the selection of parameters for the comparative algorithm. This part of the instrument, denoted by 4, is used to select one of the offered algorithms as well as to set its parameters, as described in Section 1. The reconstruction results are displayed graphically and evaluated numerically through MSE, computational time, and concentration measures. The outlook of this part of the software is shown in Figure 3. In the presented example (Figure 3), for a chosen QRS complex and 50% of available samples, the best reconstruction performance is provided by the Gradient algorithm and ℓ_1 -magic algorithms in terms of concentration measures. However, the gradient-based algorithm provides slightly better MSE.

4.2. Part 2—Reconstruction of 2D Signals

The second part of the Virtual instrument is designed for image reconstruction. Switching between two parts of the instrument can be done using the *1D/2D* button at the top left corner of the instrument. The reconstruction of images is performed by several algorithms. Radial-Fourier uses TV minimization with the 2D DFT as a transform basis. The measurements are collected along radial lines from the 2D DFT domain, while the number of radial lines is set as an input parameter of the algorithm.

TV minimization is used with the 2D DCT transform within the second algorithm from the drop-down menu, where there is a possibility to set the number of low and middle frequency coefficients. Another approach is the 2D gradient algorithm, as this algorithm produces efficient results with natural images even in a very low number of available samples. A variation on the method of multipliers is implemented through the Douglas–Rachford algorithm for biomedical image recovery (the UNLocBoX software is used for Douglas–Rachford algorithm implementation [51]).

The outlook of this part of the instrument is shown in Figure 4. Users can use some of the predefined images, or load their own images. Predefined images can be selected in Section 1. The image database is downloaded from [52].

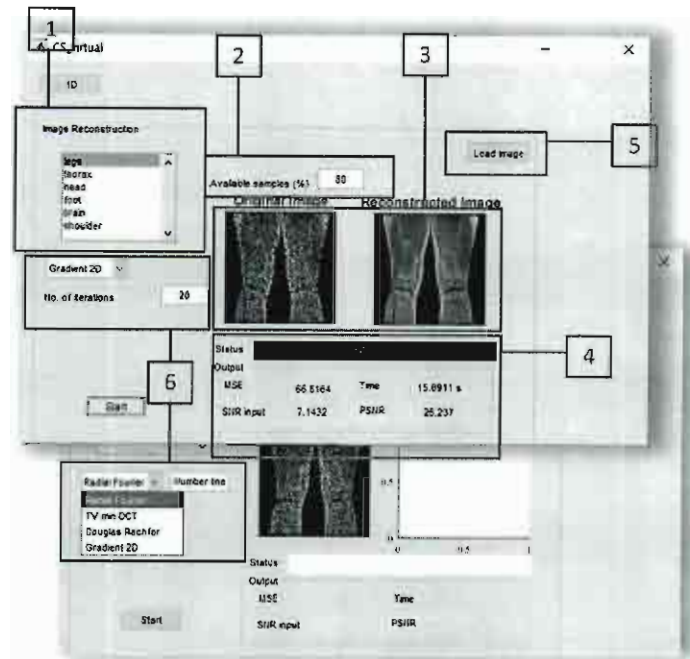


Figure 4. The outlook of 2D part of the proposed Virtual instrument.

Section 2: For the selected image, the percent of available samples is selected (i.e., a number of radial lines when considering the radial-Fourier algorithm or number of middle and/or low-frequency coefficients in the TV min approach). The gradient algorithm requires a number of iterations. Section 3: In this section, the original image with missing pixels is shown (left), and the image after reconstruction is performed is shown on the right.

Section 4: Numerical results of reconstruction: Here, MSE, computational time, and input signal to noise ratio SNR and output peak signal to noise ratio PSNR are calculated and displayed. The wait bar is implemented in order to visually show the reconstruction progress. It is important to note that the user can select the arbitrary image that needs to be located within MATLAB m-file (Section 5). Section 6 shows the selection of the algorithms.

5. Additional Experimental Evaluation

In this section, some additional reconstruction scenarios are provided and discussed.

Example 1: Firstly, the 1D signals are tested. The plots from the virtual instrument are extracted and presented within the diagram in Figure 5. Therefore, the approach that provided successful reconstruction in all considered signals, TV minimization, is used for obtaining the results in Figure 5. Signals for heart rate monitoring, the ECG and the extracted QRS complex; the respiration signal; and the signals for brain activity monitoring, the EEG and EOG signals, are tested. The signal is firstly reshaped from vector to matrix (2D) form. Reshaping is done column-wise. The DCT is considered as a domain of sparsity and the measurements are collected randomly from this domain. The next step is the reconstruction of 2D data using TV minimization, as shown in Figure 5. Finally, the reconstructed signal is back to its 1D form.

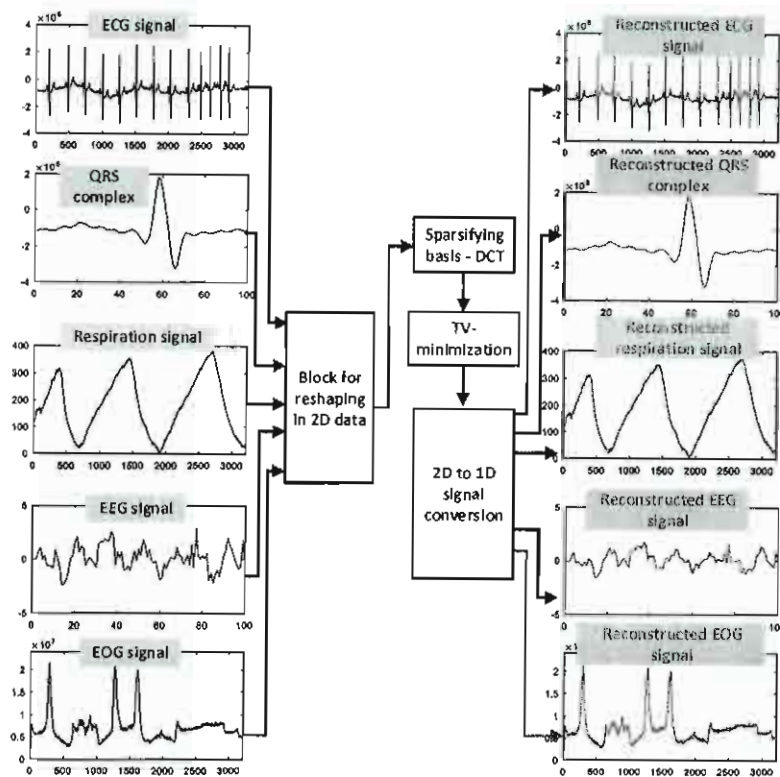


Figure 5. The reconstruction results for different 1D biomedical signals. The total variation (TV) minimization is used and 45% of samples are considered as unavailable. EEG, electroencephalography; EOG, electrooculography.

Example 2: In this example, the reconstruction efficiency is tested for the MRI image. Unlike the 1D case, the MRI image can be successfully recovered with almost all implemented algorithms. The percentage of missing samples is considered to be around 45. The original and image with missing samples are shown in Figure 6. The reconstruction procedures show that the radial-Fourier provided the best PSNR. However, the processing time is the longest using the radial-Fourier approach. The reconstruction results are shown in Table 1 and Figure 7.

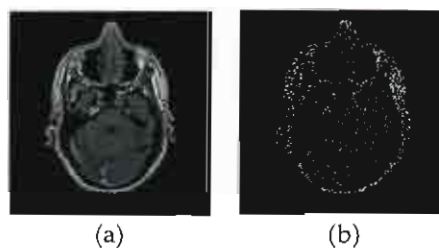


Figure 6. The original (a) and image with missing pixels (b); 45% of pixels are unavailable.

Table 1. The PSNR and reconstruction time for different algorithms. TV, total variation; DCT, discrete cosine transform.

Algorithm	Percentage of Missing Pixels	Reconstruction Time (sec)	PSNR [dB]
Gradient	45%	11.4	30.5
Radial-Fourier	43%	120.1	47.8
TV-min-DCT	45%	0.9	43.9
Douglas-Rachford	45%	11.2	31.5

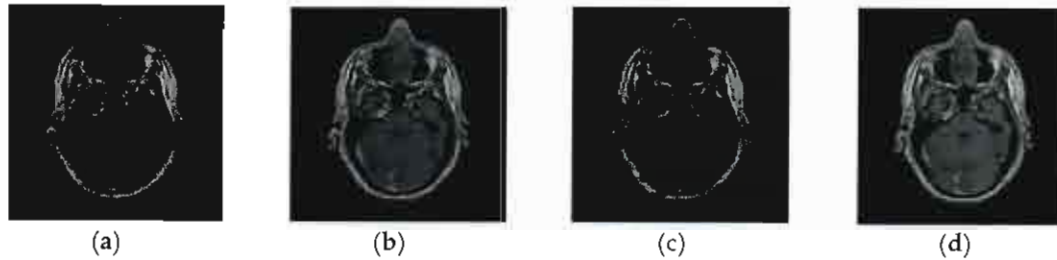


Figure 7. Reconstruction results for the magnetic resonance imaging (MRI) image, considering 45% the missing information; the results are obtained using the (a) gradient; (b) radial-Fourier; (c) TV-min-DCT; and (d) Douglas–Rachford algorithms.

6. Conclusions

Virtual instrument for compressive sensing signal reconstruction is proposed. The software is composed for a specific purpose, that is, biomedical signal recovery, considering both 1D and 2D biomedical signals. Several commonly used algorithms for sparse signal recovery are implemented. Additionally, the proposed instrument enables a comparison of different algorithms, where specific parameters can be changed independently for each algorithm. The second part of the instrument is used for image reconstruction. All functions within the instrument can be used, upgraded, or changed with some other algorithms in order to build other application-related instruments for solving specific problems. This software can be a useful tool for clinicians and healthcare professionals in an era when low-power portable medical devices become widely used and when safe and fast communication is of great interest. The part enabling the comparison of algorithms and choosing the most suitable one can be useful in medical practice, as it enables selection of the most accurate and fastest approach.

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Adaptive variable step algorithm for missing samples recovery in sparse signals

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Abstract: Recovery of arbitrarily positioned samples that are missing in sparse signals recently attracted significant research interest. Sparse signals with heavily corrupted arbitrary positioned samples could be analysed in the same way as compressive sensed signals by omitting the corrupted samples and considering them as unavailable during the recovery process. The reconstruction of the missing samples is done by using one of the well-known reconstruction algorithms. In this study, the authors will propose a very simple and efficient algorithm, applied directly to the concentration measures, without reformulating the reconstruction problem within the standard linear programming form. Direct application of the gradient approach to the non-differentiable forms of measures lead us to introduce a variable step size algorithm. A criterion for changing the adaptive algorithm parameters is presented. The results are illustrated on the examples with sparse signals, including approximately sparse signals and noisy sparse signals.

1 Introduction

In many signal processing applications, a signal that spans over the whole time domain is located within much smaller regions in a transformation domain. If we consider a discrete time-limited signal, it could contain much fewer non-zero samples (coefficients) in an arbitrary transformation domain (Fourier domain, discrete cosine domain, discrete wavelet domain etc.). The signal is then sparse in this transformation domain. If this condition is satisfied, we can reconstruct the signal without using the whole dataset required by the Shannon–Nyquist sampling theorem. Processing of the sparse signal with a large number of missing/unavailable samples has attracted significant interest in recent years. This research area interacts with many other research areas such as signal processing, statistics, machine learning and coding. Compressive sensing/sampling (CS) is a field dealing with sparse approximations [1, 2]. The crucial parameter in the approximation is the number of available samples/measurements used in the reconstruction. It is directly related to the number of non-zero coefficients in the sparse domain [1–3].

The signal samples may be missing because of their physical or measurements unavailability. Also, if some arbitrary positioned samples of a signal are heavily corrupted by a disturbance, it has been shown that it is better to omit than to use them in the analysis or processing (by L-estimation, e.g. [4–6]). Both these situations correspond to the CS approximation problems, if the analysed signal is sparse. Under the conditions defined within the CS framework, the processing of such a signal could be performed with the remaining samples, almost as

in the case if all the missing/unavailable samples were available.

Several approaches to reconstruct these kinds of signals are introduced [7–26]. One group is based on the gradient [7] and the other is based on the matching pursuit approaches [25]. A common approach to this problem is based on redefining it within the linear programming (LP) as the bound constrained quadratic program (BCQP). A measure of the signal sparsity is used as a minimisation function in the sparse signal reconstruction. This measure is related to the number of non-zero transformation coefficients. This kind of measure was also used, especially in the time-frequency analysis, for measuring the concentration of a signal representation. Since, the sparsity of a signal in a transformation domain is related to the number of non-zero samples, a natural mathematical form to measure the number of non-zero (significant) samples in a signal transform is the so called norm-zero (l_0 norm). It is a sum of the signal transformation absolute values raised to the zeroth power. Since this power produces value one for any non-zero transformation coefficient, it just counts the number of non-zero coefficients. However, this norm is very sensitive to any kind of disturbance that can make the original zero transformation coefficients small but different from zero. Thus, more robust norms are used. The norm that may be used for measuring the transformation concentration, being also less sensitive to the disturbances, is the norm-one (l_1 norm). Since the norm-one is not differentiable around the optimal point, the CS algorithms reformulate the problem under the norm-two (l_2 norm) conditions before the optimisation is done.

In this paper, we will present a gradient based algorithm for the reconstruction of the sparse signals. The presented

algorithm uses an arbitrary concentration measure in a direct way, without redefining the problem to a quadratic form and by using linear programming tools. Since the signal reconstruction is required in the time domain, the proposed algorithm performs a search over the missing samples values in the time domain. The presented algorithm can reconstruct a large number of missing samples in a computationally efficient way. The proposed method belongs to the class of the gradient based CS algorithms [10]. However, common adaptive signal processing and the CS algorithms avoid the direct use of the measure based on the norm-one (or similar norms between norm-zero and norm-one) since it is not differentiable. The intensity of the gradient cannot be used as an indicator of the proximity of the iteration values to the algorithm solution. When the iterations are close to the optimal point, the gradient intensity remains constant in the norm-one case. Taking a sufficiently small step over the whole range would not be a solution, because of an extremely large number of iterations over a very large set of variables. Here, the adaptive gradient-based approach is directly applied to an appropriately chosen concentration measure. Since for the commonly used norm-one (or any other norm between norm-zero and norm-one) based concentration measure the derivatives are not continuous functions around the minimum, a variable and self-adaptive step in the algorithm is introduced. The presented algorithm, with this adaptive step, reconstructs a large number of missing samples in a simple and computationally efficient way with an arbitrary (or computer defined) precision of the results.

The paper is organised as follows. After the introduction, a review and analysis of the concentration measures in the processing of sparse signals is done. A gradient-based algorithm, with its modifications, is presented and illustrated. The presented algorithm's efficiency is demonstrated on several examples with a large number of missing samples, including the samples missing in the blocks and the noisy signals. The basic idea for this algorithm was presented in [27].

2 Measures and direct reconstruction

The concentration measures of the signal transforms were intensively studied and used in the area of time–frequency signal analysis and processing. They are used to find an optimal, best concentrated time–frequency representation of a signal. The most common and the oldest measure, introduced to measure the concentration of the time–frequency representations, is defined by Jones, Parks, Baraniuk, Flandrin, Williams *et al.* The concentration of a signal transform $X(k)$ is measured by

$$\mathcal{M}^{(4/2)}[X(k)] = \frac{\sum_k |X(k)|^4}{\left(\sum_k |X(k)|^2\right)^2} \quad (1)$$

In general, it has been shown that any other ratio of norms

$$l_s = \left(\sum_k |X(k)|^s \right)^{1/s} \quad \text{and}$$

$$l_q = \left(\sum_k |X(k)|^q \right)^{1/q}, \quad s > q > 1$$

can also be used for measuring the concentration. These kinds of concentration measures were inspired by the kurtosis as a measure of the distribution peakedness. Similar forms are obtained by using the Rényi measures.

Another direction to measure the time–frequency representation concentration comes from a classical definition of the time-limited signal duration, rather than measuring the signal peakedness. It was used in the time–frequency analysis in [24]. If a signal $x(n)$ is time-limited, that is, $x(n) \neq 0$ only for $n \in [n_1, n_2 - 1]$, then the number of non-zero values in $x(n)$ is $d = n_2 - n_1$. It can be obtained as

$$d = \lim_{p \rightarrow \infty} \sum_n |x(n)|^{1/p} = \|x(n)\|_0 \quad (2)$$

where $\|x(n)\|_0$ denotes the so called norm-zero l_0 of the signal. In reality, there is no sharp edge between $x(n) \neq 0$ and $x(n) = 0$, so the value of d in (2) could, for very large p (close to norm-zero), be sensitive to the small values of $|x(n)|$. The robustness may be achieved by using lower-order forms, with $1 \leq p < \infty$ (norms from l_1 to l_0).

Therefore the concentration of a signal transform $X(k) = \mathcal{T}[x(n)]$ can be measured with the function of the form

$$\mathcal{M}_p[\mathcal{T}[x(n)]] = \frac{1}{N} \sum_k |X(k)|^{1/p} \quad (3)$$

with $1 \leq p < \infty$, corresponding to the norm $l_{1/p} = \left(\sum_k |X(k)|^{1/p} \right)^p$, where N is the total number of samples in the signal transform $X(k)$. A lower value of (3) indicates a better concentrated distribution. For $p = 1$, it is the norm-one form

$$\mathcal{M}_1[\mathcal{T}[x(n)]] = \frac{1}{N} \sum_k |X(k)| = \frac{1}{N} \|X(k)\|_1$$

Minimisation of the norm-one of the short-time Fourier transform (corresponding to the norm l_2 of the spectrogram) is used in [24] to optimise the window width and to produce the best concentrated signal representation. The norm-one is also the most commonly used in the CS algorithms for measuring the signal sparsity/concentration [1, 2, 17, 18]. Here, we will illustrate the influence of the measure parameter p on the results, including the explanation why the norms greater than one ($p < 1$, including l_2 case) cannot be used to measure the concentration. These norms could be used in the ratio forms (1) only, but not as the stand-alone transformation measures, as in the case of the $1 \leq p < \infty$ measures.

The simplest reconstruction algorithm will be based on a direct search over all the unavailable/missing samples values, by minimising the concentration measure. If we consider a complete set of signal samples $\{x(1), x(2), \dots, x(N-1)\}$ and M samples $x(m_1), x(m_2), \dots, x(m_M)$ are missing then the simplest algorithm will be to search over all the possible values of the missing samples and find the solution that minimises the concentration measure

$$\min_{x(m_1), x(m_2), \dots, x(m_M)} \left\{ \mathcal{M}_p[\mathcal{T}[x(n)]] \right\}$$

From the remaining samples, we can estimate the range limits for the missing samples, $|x(m_k)| \leq A$. In the direct search approach, we can vary each missing sample value from

– A to A with a step $2A/(L-1)$ where L is the number of considered values within the selected range. It is obvious that the reconstruction error is limited by the step $2A/(L-1)$ used in the direct search. The number of the analysed values for the M coefficients is L^M . Obviously, the direct search can be used for a small number of missing samples only, since for any reasonable accuracy the value of L is large.

One possible approach to reduce the number of operations in the direct search is to use a large step (small L) in the first (rough) estimation, then to reduce the step around the rough estimate of the unavailable/missing values $x(m_1), x(m_2), \dots, x(m_M)$. This procedure can be repeated several times, until the desired accuracy is achieved. For example, for $A=1$, an accuracy of 0.001 is achieved in one iteration if $L=2001$ with, for example, seven missing samples that would mean an unacceptable number of $2001^7 \sim 10^{23}$ measure calculations. However, if the first search is done with, for example, $L=5$, the rough optimal values are found, and the procedure is repeated with the $L=5$ values within the range determined by the rough optimal in the first step. By repeating the same procedure six more times, an accuracy better than 0.001 is reached with $7 \times 5^7 \sim 10^5$ measure calculations. In this way, we have been able to analyse (on an ordinary PC, within a reasonable calculation time) signals with up to ten missing samples.

Although computationally not efficient, the direct search method is very important and helpful in the analysis of various concentration measures with different p , since the more advanced and efficient methods from the literature produce results with nice values of p only (e.g. $p=1, p=1/2$ or $p=2$). The direct method can be used with any p . Also, the probability that we find and stay in a local minimum is lower in the direct search method than when using the other algorithms. Thus, we will use the direct search to illustrate how the solution depends on the chosen norm (concentration measure form).

Example: Consider a discrete signal

$$x(n) = 2.5 \sin(20\pi n/N) \tag{4}$$

for $n=0, 1, \dots, N-1$, and $N=256$ is the number of the signal samples. The case of the two missing samples is presented first, as the one appropriate for the graphical illustration. The direct search is performed over the range $[-5, 5]$ with a step 0.01. Measure (3) is calculated for various values of the parameter p . The results are shown in Fig. 1. The measure minimum is located on the true sample values for $p \geq 1$ (norms l_1 and lower). The measure minimum for $p < 1$ (including norm-two, for $p=1/2$) is not located at the true signal values. An important case with the two missing samples and $p=1$ is presented in Fig. 2.

To illustrate the measure influence on the mean absolute error (MAE) the direct search is also performed on the signal

$$x(n) = 3 \sin(10\pi n/N) + 2 \cos(30\pi n/N) \tag{5}$$

This signal is composed of $N=256$ samples while the cases with the four and seven missing samples are analysed. The results with 10 and 15 iterations (to reduce the step size) are presented in Fig. 3. We can see that:

(1) $p \geq 1$ (norms l_q with $q=1/p \leq 1$, including l_1) produces accurate results with the MAE depending on the direct

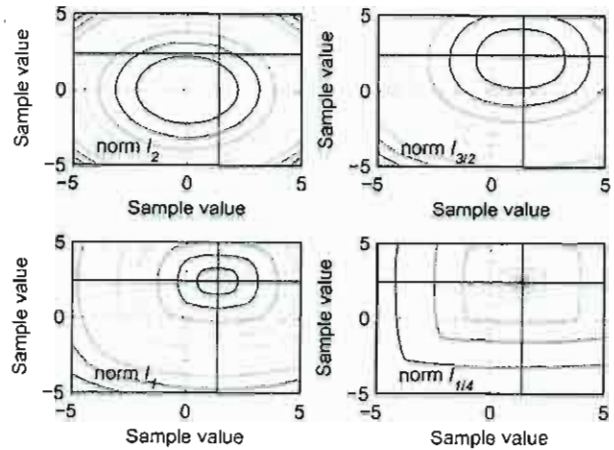


Fig. 1 Measure as a function of the two missing sample values corresponding to various norms

True values of the missing samples are presented with straight lines

search step only. The MAE can be further reduced to the computer precision by an iterative reduction of the step size. (2) For $p < 1$ (norms l_q with $q > 1$, including l_2) the bias dominates over the number of iterations, so that the results are almost independent from the number of iterations.

Almost the same results are obtained for the four and the seven missing sample cases.

The minimisation using the l_2 norm: For $p=1/2$ this measure is equivalent with the well-known l_2 norm used in the definitions of the standard signal transforms [6]. For the norm-two (l_2 norm with $p=1/2$) the MAE is of the signal samples order, as shown in Fig. 3. The measure with the l_2 norm has a minimum when the missing signal samples are set to zero.

This result was expected and can be proven for any number of missing samples for the signals and its transforms satisfying Parseval's theorem. Parseval's theorem states that the energy of a signal in the time domain is the same as the energy of the Fourier transform in the frequency domain. We know that a signal has the lowest energy when its missing samples are zero-valued. Associating any

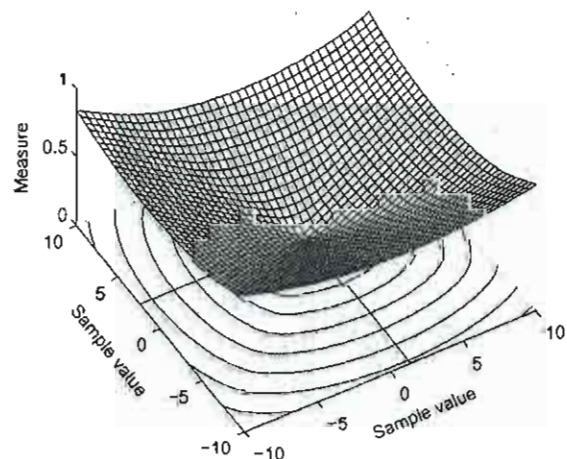


Fig. 2 Measure for $p=1$ (norm l_1) as a function of the missing samples values

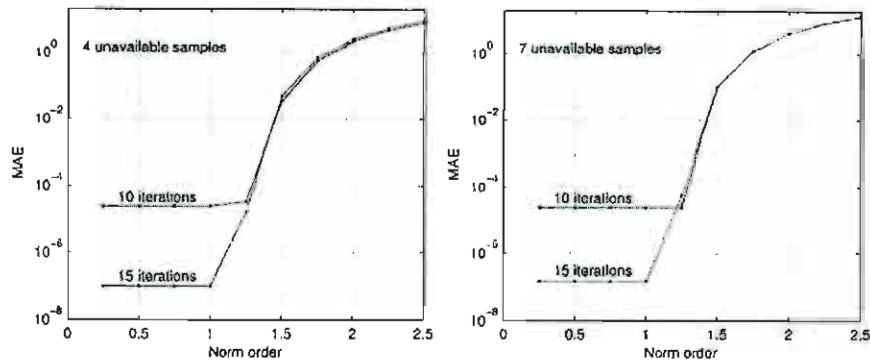


Fig. 3 MAE in the coefficients estimation as a function of the norm (measure) order for the four missing samples (left) and the seven missing samples (right)

The MAE is normalised with the number of missing samples

non-zero value to the missing samples will increase the signal energy. The same holds in the frequency domain since the energy in the frequency domain equals to the energy in the time domain. The minimisation solution with the l_2 norm is therefore trivial. With this norm, we attempt to minimise

$$\|X\|_2^2 = \sum_{k=0}^{N-1} |X(k)|^2$$

According to Parseval's theorem we have $\|X\|_2^2 = N \sum_{n=0}^{N-1} |x(n)|^2$. Since any value other than $x(n)=0$ for the unavailable/missing signal samples would increase $\|X\|_2^2$, then the solutions for the non-available samples, with respect to the l_2 norm, are all zero values. This was the reason why this norm was not used as a concentration measure. This is also the reason why this norm cannot be used in the CS-based algorithms for the missing samples recovery.

Therefore, in theory, the most obvious case for the concentration measures is for $p \rightarrow \infty$ when the norm l_0 is used. This value counts exactly the number of the non-zero coefficients (sparsity), regardless of their value. However, this norm is not applicable in practice since it is extremely sensitive to any kind of disturbance, even to a computer precision error. Also, the gradient based algorithms cannot be used with this norm. The norms with $p < 1$ produces biased results. We have proven that, for example, the norm l_2 (with $p=1/2$) would be minimal when all the missing samples are set to zero. Thus, the value $p=1$ is an obvious and simple choice and it is most commonly used in the literature. The restricted isometry property is used to prove that under some conditions this norm produces the same result as l_0 [1, 2]. Some improvements may be achieved by using values just slightly lower than $p=1$. Then, the results will be slightly biased, but the norm will be differentiable. It can improve the gradient-based algorithms performance, especially in a few starting iterations.

3 Adaptive gradient-based algorithm

Owing to a high computational complexity, the direct search could be used only if the number of missing samples M is small enough. It is the reason why many other, more sophisticated, CS algorithms have been proposed. Here, we will present one very simple and efficient algorithm, based on the direct use of the concentration measure gradient.

This algorithm is inspired by the adaptive signal processing methods with a variable step size. This algorithm is a gradient descent algorithm where the missing samples are estimated as the ones producing the minimal concentration measure of the signal transform in the sparse domain.

The norms that produce the unbiased missing samples values (such as, e.g. norms with $p \geq 1$) are not differentiable around the optimal point. It means that the gradient method, if directly applied to the measure based on, for example, the l_1 norm (or any other norm with $p \geq 1$) will have a problem when approaching the optimal point. Since the gradient intensity in the vicinity of the optimal point is almost constant for $p=1$, the algorithm will not improve the accuracy to a level lower than the accuracy defined by the step in the gradient algorithm. This is the reason why this approach has not been used and why appropriate reformulation of this problem is done in the literature. These reformulations are done within the linear programming by using the well-known and available norm-two-based solutions. Here, we will not try to reformulate the problem based on the l_1 norm within the linear programming l_2 framework, but we will use the gradient-based adaptive algorithm, with the step being appropriately adjusted (in a simple way) around the optimal point. The algorithm presented here will be a simple and efficient application of the gradient-based adaptive approach to the measures that are not differentiable around the optimal point.

As we can see from Fig. 2, the measure with $p=1$ is differentiable and convex everywhere except around the point of the minimum (the optimisation solution point). Therefore, any algorithm applied directly to the measure based on $p=1$ will oscillate around the solution with an amplitude defined by the step and the measure form (this will be illustrated within the examples). If we take a very small step for each of a large number of missing samples, it will result in an unacceptable and large number of iterations. Thus, when the steady oscillatory state in the measure function is detected we should reduce the algorithm step, as we presented in the direct search. In this way, the results with a high accuracy, within an acceptable number of iterations, are achieved by using a variable self-adaptive step. This simple method is able to produce the results with an error of the computer precision level. Finally, in addition to the step variation, these kinds of algorithms enable the parameter p (the norm form itself) to change to improve the initial convergence of the algorithm.

3.1 Algorithm

Consider a discrete signal $x(n)$ with some samples that are not available. Assume that the signal is sparse in a transformation domain $T[x(n)]$. The algorithm for the missing samples reconstruction is implemented as follows:

Step 0: Form the initial signal $y^{(0)}(n)$, where (0) means that this is the first iteration of the algorithm, defined as

$$y^{(0)}(n) = \begin{cases} x(n) & \text{for available samples} \\ 0 & \text{for missing samples} \end{cases}$$

Step 1: For each missing sample at n_i we form two signals $y_1(n)$ and $y_2(n)$ in each next iteration as

$$y_1^{(k)}(n) = \begin{cases} y^{(k)}(n) + \Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

$$y_2^{(k)}(n) = \begin{cases} y^{(k)}(n) - \Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

where k is the iteration number. Constant Δ is used to determine whether the considered signal sample should be decreased or increased.

Step 2: Estimate the differential of the signal transform measure as

$$g(n_i) = \frac{\mathcal{M}_p[T[y_1^{(k)}(n)]] - \mathcal{M}_p[T[y_2^{(k)}(n)]]}{2\Delta} \quad (6)$$

where \mathcal{M}_p is defined by (3). The differential of the measure is proportional to the error ($y^{(k)}(n) - x(n)$).

Step 3: Form a gradient vector G with the same length as the signal $x(n)$. At the positions of the available samples, this vector has value $G(n)=0$. At the positions of the missing samples its values are $g(n_i)$, calculated by (6).

Step 4: Correct the values of $y(n)$ iteratively by

$$y^{(k+1)}(n) = y^{(k)}(n) - \mu G(n)$$

where μ is a step size that affects the performances of the algorithm (the error and the speed of convergence).

By repeating the presented iterative procedure, the missing values will converge to the true signal values which produce a minimal concentration measure in the transformation domain. The algorithm performance depends on the parameters μ and Δ .

3.2 Varying and adaptive step size

Since we use a difference of the measures to estimate the gradient, when we approach the optimal point, the gradient with norm l_1 will be constant and we will not be able to approach the solution with a precision higher than the step μ , multiplied by a constant (gradient dependent) value. If we try to reduce the oscillations around the true value by using a smaller step from the beginning, then we will be faced with an unacceptable number of iterations. However, this problem may be solved, by reducing the step size, when we approach the stationary oscillations zone. The best solution is to use an adaptive step size in the algorithm. A large step size should be used when the concentration measure is not close to its minimum (in the starting

iterations). The step is reduced as we approach the concentration measure minimum. Next, we will present a method for the adaptive parameters adjustment that could be applied to the algorithm in order to reduce the error and increase the accuracy.

When the algorithm with the constant parameters is close to the optimal point, the concentration measure tends to have constant value in a few consecutive iterations. This behaviour will be detected by checking the difference between two consecutive measure calculations $\mathcal{M}_p^{(k-1)} - \mathcal{M}_p^{(k)}$, where $\mathcal{M}_p^{(k)} = \mathcal{M}_p[T[y^{(k)}(n)]]$ is the sparsity measure of the reconstructed signal in the k th iteration. When it is smaller than, for example, 1% of the highest previously calculated measure difference

$$\mathcal{M}_p^{(k-1)} - \mathcal{M}_p^{(k)} \leq P \max_{m=1,2,\dots,k-1} |\mathcal{M}_p^{(m-1)} - \mathcal{M}_p^{(m)}| \quad (7)$$

where $P=0.01$, the algorithm parameters Δ and μ should be reduced, for example, by ten times.

4 Numerical examples

Consider the signal

$$x(n) = 3 \sin(20\pi n/N) + 2 \cos(60\pi n/N) + 0.5 \sin(110\pi n/N) \quad (8)$$

The total number of the signal samples is $N = 256$. We assume that 200 samples (80% of the total number of samples) are missing or are not available. Two cases will be considered. One, when the missing samples are randomly positioned and the other when the samples are missing in randomly positioned blocks. We know their positions, and also that the signal is sparse in the Fourier domain. Here, we will perform the signal reconstruction with the constant algorithm parameters $\Delta = 2$, $\mu = 3$ and $p = 1$.

The reconstruction results are shown in Figs. 4 and 5. Since the constant algorithm parameters are used, the achieved error is small but still notable, Figs. 4d and 5d. The residual error value is determined by the algorithm parameters. Next, we will analyse the influence of the parameters Δ , μ and p on the number of iterations and the mean absolute error (MAE). The MAE in the k th iteration, calculated as

$$\text{MAE}(k) = \frac{1}{N} \sum_n |x(n) - y^{(k)}(n)|$$

is shown in Fig. 6 for various algorithm setups. It can be concluded that for the constant algorithm parameters, the MAE cannot be improved by increasing the number of iterations below some limit. Smaller values of Δ and μ produce lower MAE but with an increased number of iterations, as presented in Figs. 6a and b. The results obtained for varying Δ and μ are presented in Fig. 6c. Here, the parameters are changed at the iterations $k=100$ and $k=200$. We can see that with the same number of iterations, a smaller MAE is achieved. Therefore, the parameters Δ and μ should be adaptive, resulting in the MAE presented in Fig. 6d. Here, we detect that, after a number of iterations, the gradient algorithm does not further improve the sparsity of the reconstructed signal. Then, we use the smaller values of Δ and μ for the next iterations, as described in the previous section. In Figs. 6e and f, the absolute errors in

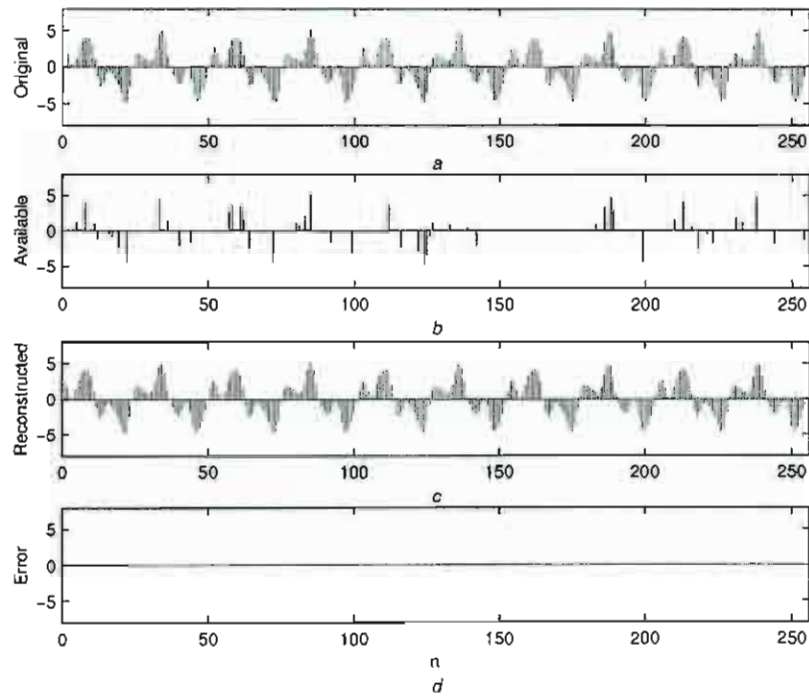


Fig. 4 Reconstruction example for a signal with 200 missing samples at random positions
a Original signal
b Signal with missing samples set to 0 and used as an input to the reconstruction algorithm
c Reconstructed signal
d Reconstruction error

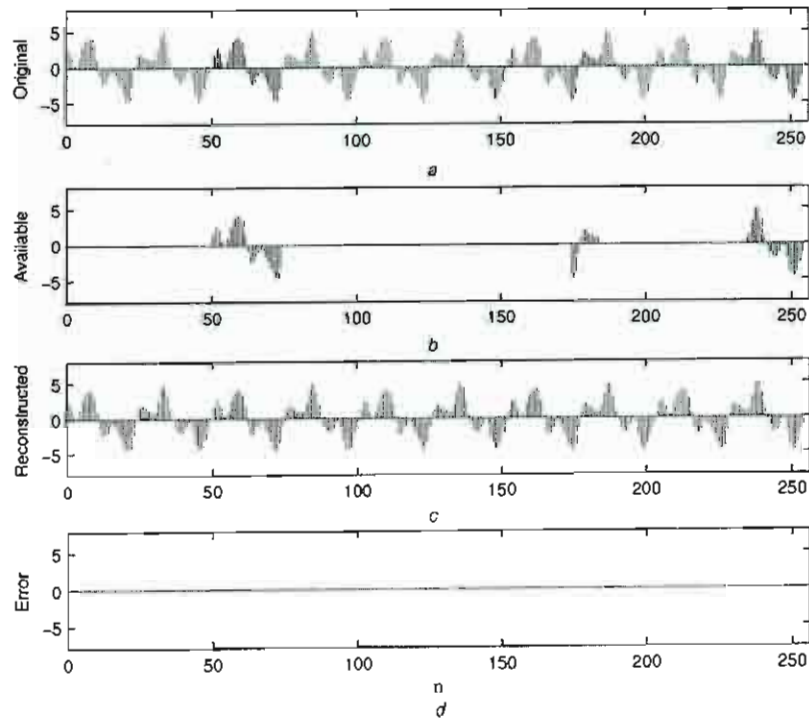


Fig. 5 Reconstruction example for a signal with 200 missing samples grouped into three blocks
a Original signal
b Signal with missing samples set to 0 and used as an input to the reconstruction algorithm
c Reconstructed signal
d Reconstruction error

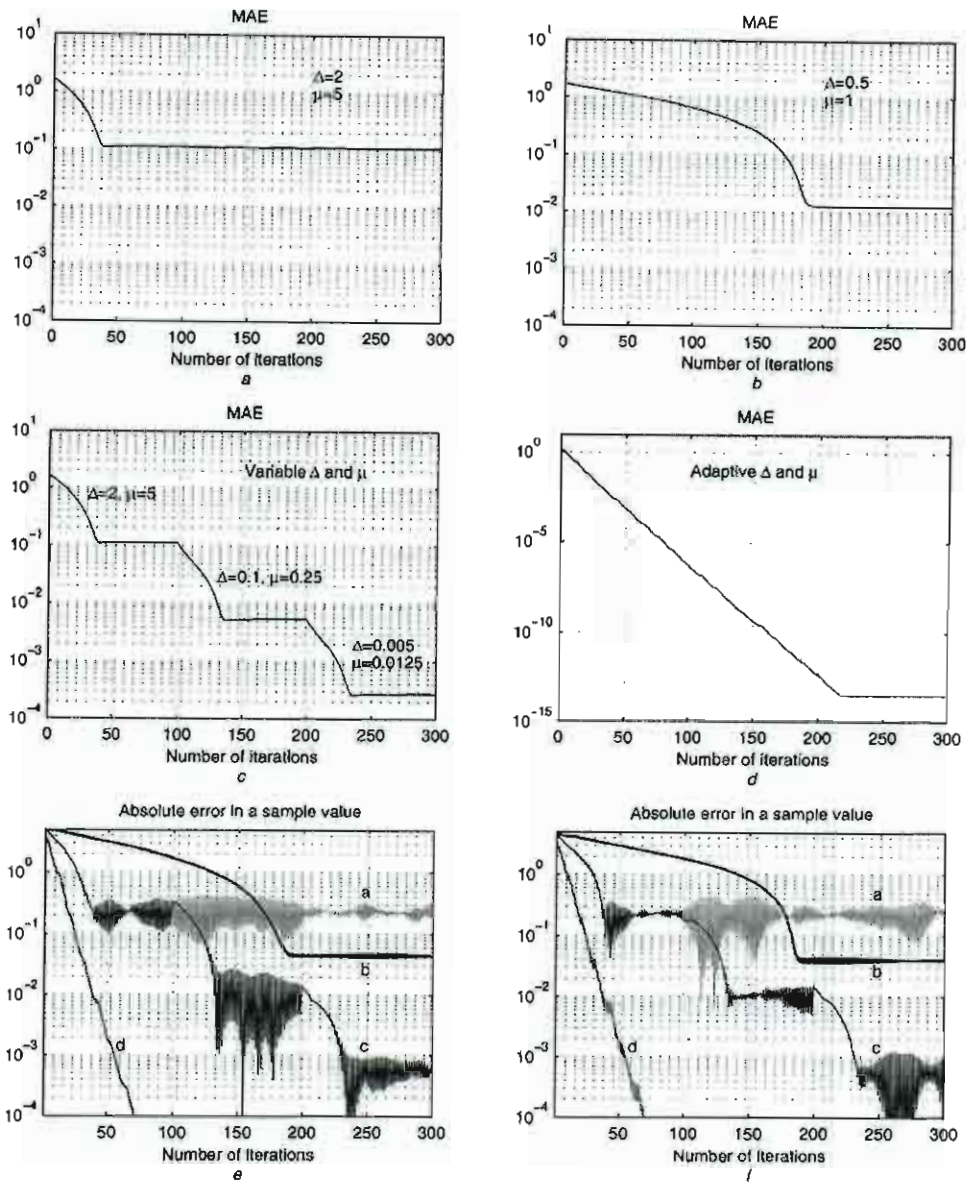


Fig. 6 MAE for the constant algorithm parameters Δ and μ

a and *b* Variable parameters and adaptive parameters

c and *d* Absolute errors for the two randomly chosen missing signal samples

e and *f* For the constant algorithm parameters (lines *a* and *b*), variable (line *c*) and adaptive parameters (line *d*)

two signal samples, during the iteration process, are shown for all the previous cases of the algorithm setup. It can be seen that the absolute errors behave in a similar manner as the MAE in the above subplots, with the difference that they oscillate around the steady value, because of the non-differentiable measure around the solution.

Adaptive algorithm parameters: From the previous analysis and the theoretical considerations, we have concluded that the adaptive algorithm parameters should be used. The results obtained by the proposed method for a self-adaptive parameter adjustment are presented in Fig. 7. Criterion (7) for the adaptive step size is applied on signal (8), with 150 randomly positioned missing signal samples. The starting parameters for the adaptive algorithm were $\Delta=20$ and $\mu=20$. The graphics in Fig. 7*a* illustrates the MAE as a function of the iteration number. Each line on this graph matches one set of the parameters Δ and μ . The

parameter values were divided by ten (and the line in the graph is changed) when the condition (7) is met.

The dashed lines represent the MAE when the algorithm with constant parameters is run from the same initial point. Line *a* is for the MAE when the constants $\Delta=20$ and $\mu=20$ are used. Line *b* is for the MAE when $\Delta=20$ and $\mu=20$ are used at the beginning, whereas the algorithm has changed the parameters to $\Delta=2$ and $\mu=2$ when condition (7) is met. The dashed line *b* represents the MAE if $\Delta=2$ and $\mu=2$ are used from the first iteration. Line *c* is for the MAE when the algorithm parameters at the beginning were $\Delta=20$ and $\mu=20$, then changed to $\Delta=2$ and $\mu=2$, and finally changed to $\Delta=0.2$ and $\mu=0.2$. The dashed line *c* represents the MAE if $\Delta=0.2$ and $\mu=0.2$ were used from the first iteration. This process continues in the same way 2 more times for Figs. 7*a* and 12 more times for Fig. 7*b*.

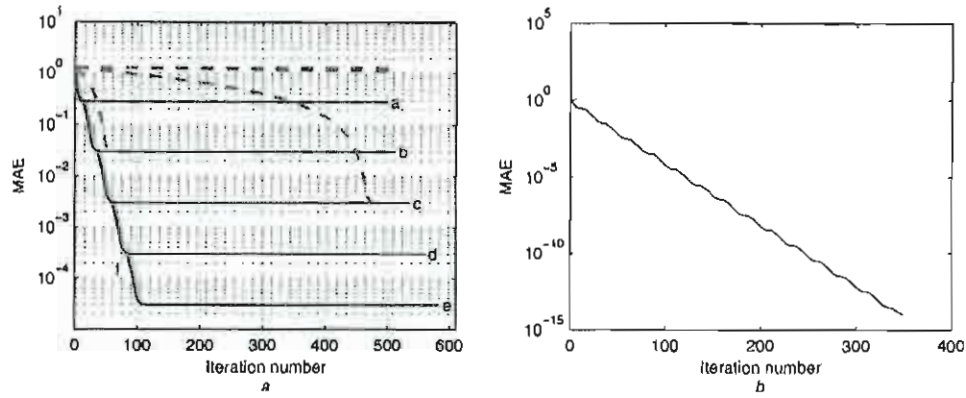


Fig. 7 MAE for the adaptive parameters using the measure-based criterion

Each labelled line on the graphics

a Presents one set of parameters Δ and μ . Thick grey line presents the MAE with the adaptive parameters

b MAE is presented for the case when the algorithm parameters adaptation is done up to the computer precision

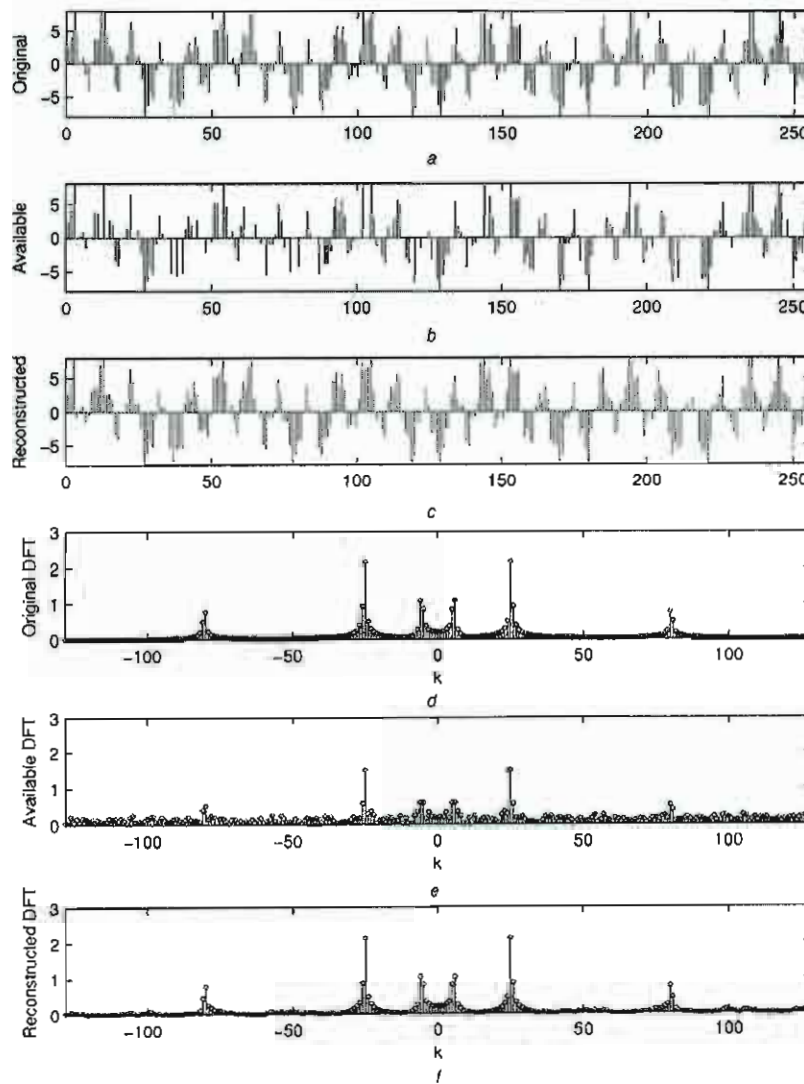


Fig. 8 Reconstruction of an approximately sparse signal whose frequencies do not match the frequency grid

a Original signal

b Available signal

c Reconstructed signal

d DFT of the original signal

e DFT of the available signal

f DFT of the reconstructed signal

Note that from the behaviour of the dashed lines (constant parameters) we can conclude that they achieved their stationary state in ten times more iterations than by using the previous larger parameters. The dashed line b achieved the stationary MAE in about 50 iterations and dashed line c in about 500 iterations. Hence, we can expect that the dashed line d will achieve its stationary MAE in about 5000 iterations and so on.

We may conclude that the MAE would achieve a 10^{-15} value (what is the standard computer double precision error) in about 10^{14} iterations with the corresponding constant parameters. Of course, this is not acceptable in the practical calculations. As we can see, the same order of the MAE is achieved by the presented adaptive algorithm in a relatively small number of iterations (about 350).

The performances of the proposed algorithm are compared with the l_1 -magic algorithm. We have run the l_1 -magic algorithm, with the default parameters, by using the signal from this example. The recovery result with 50 missing samples is obtained in 0.35 s, with the relative MAE = 0.0027. Then, we set the number of iterations in our algorithm to take about the same calculation time. We achieved an error of MAE = 3.8×10^{-14} with the proposed adaptive algorithm. For the 100 missing samples, the l_1 -magic produced results with MAE = 0.08, whereas the presented algorithm, within the same calculation time, produced results with MAE = 0.01. The l_1 -magic, with the default parameters, stopped calculation after achieving the results with these precisions. Our algorithm would always be able to produce the results with a computer precision error of 10^{-14} order, if not stopped by the user. The proposed algorithm produced accurate recovery with up to about 80% of the missing samples. This limit is slightly better than the l_1 -magic algorithm (about 75%).

Reconstruction of the approximately sparse signals:
Consider the signal:

$$x(n) = 3 \sin(11.2\pi n/N) + 5 \sin(50.6\pi n/N) + 2 \cos(160.8\pi n/N) \quad (9)$$

whose frequencies do not match with the frequency grid in the DFT. By definition, this signal is not sparse in the common DFT domain. We will apply the presented gradient algorithm with $\Delta=3$ and $\mu=4$ on this signal, when 70 signal samples are missing. Although the analysed signal is not sparse in a strict sense, satisfactory reconstruction results are obtained. Figs. 8a–c present the original signal, the available signal samples and the reconstructed signal, respectively. In Figs. 8d–f, the DFT coefficients of the original signal, the available samples and the reconstructed signal are shown, respectively. As we can see, although the original signal is not sparse (since its frequencies do not match with the frequency grid), the reconstruction is good.

Noisy signals: The proposed algorithm is used for the analysis of a noisy signal. It has been assumed that a sparse signal (8) is corrupted by an additive Gaussian noise. From the reconstruction based on a limited number of samples, we can come to a conclusion that in the case of the sparse noisy signals certain improvement can be achieved if a number of signal samples are omitted and the reconstruction is performed. Reconstruction for the 200 omitted samples is presented in Fig. 9.

When we omit some of the noisy samples of a sparse signal, the algorithm will try to reconstruct these samples in such a way as to keep high sparsity. It means that the goal of the algorithm is to keep the influence of noise, that is not sparse, as low as possible. A modest improvement of 2 dB

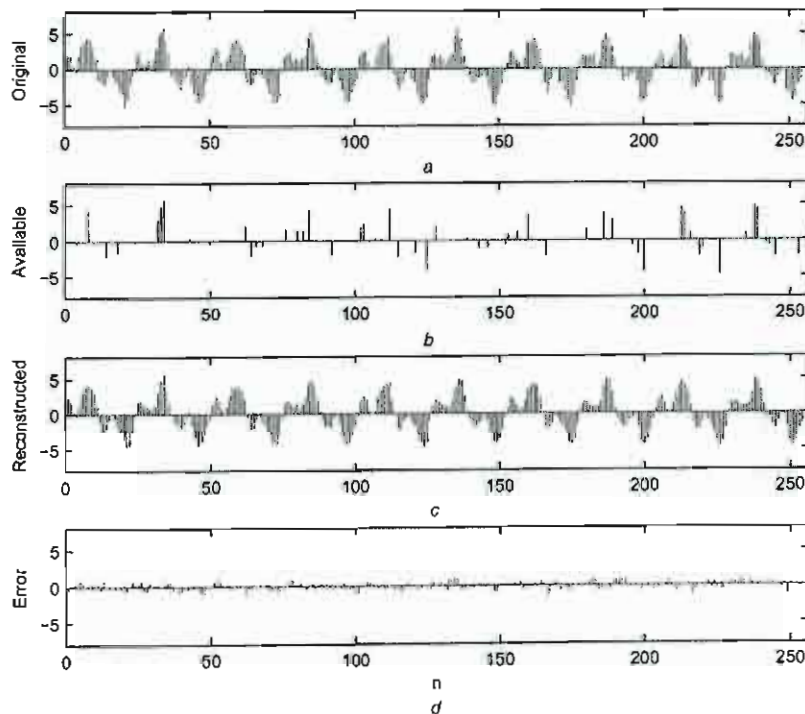


Fig. 9 Reconstruction example for a noisy signal with 200 omitted samples

- a Original signal
- b Signal with omitted samples set to 0 and used as an input to the reconstruction algorithm
- c Reconstructed signal
- d Reconstruction error (residual noise)

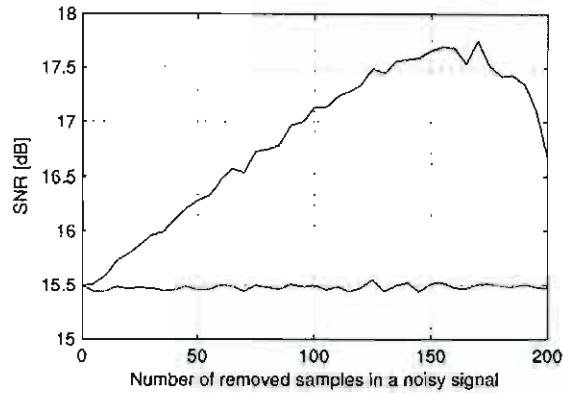


Fig. 10 SNR of the reconstructed signal as a function of the number of omitted samples (upper line). SNR of the original signal (lower line).

is achieved in average when we randomly remove about 150 out of the 256 samples, as shown in Fig. 10. Obviously, the remaining 106 samples were sufficient to reconstruct the signal. Some of the noise that was removed with the omitted samples is not reintroduced (in average). Similar results could be expected for other recovery algorithms. In this example, the signal samples, corrupted by a Gaussian noise, are removed randomly. The improvement could be higher if we were able to selectively remove the most corrupted samples. A significant improvement may be achieved in the case of the impulse noise, in combination with a sophisticated tool for noisy samples removal, such as, for example, the L-statistics [5]. Then, the most corrupted samples are removed along with a significant amount of the noise energy. Selective removal of the noisy samples is the topic for future work.

Varying concentration measure: The number of iterations for the required accuracy can be further improved by varying the measure parameter p . The measures for $p < 1$ are more suitable to the gradient-based reconstruction. However, the measures for $p < 1$ do not converge to the true values of the missing samples. A possible solution is to use measures with p slightly lower than 1 at the beginning of the iterative algorithm and to switch to $p = 1$ afterwards. Fig. 11 illustrates the case when $p = 0.9$, $\Delta = 1$ and $\mu = 10$ are used for the iterations from 1 to 12, $p = 0.95$, $\Delta = 2$ and $\mu = 4$ are used for the iterations 13–22, and finally $p = 1$,

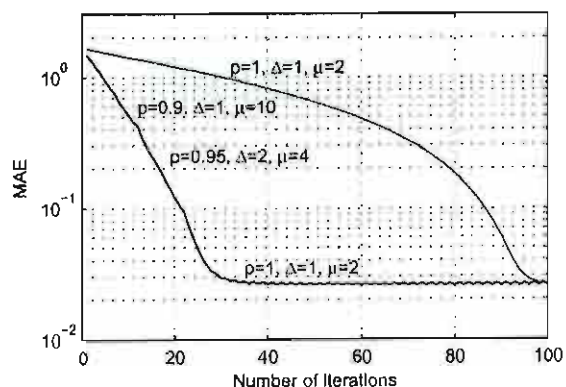


Fig. 11 Reconstruction MAE for the constant and the varying measure and the other algorithm parameters

$\Delta = 1$ and $\mu = 2$ are used for the iterations from 23 to 100, with the signal defined by (8). The case with the constant parameters $p = 1$, $\Delta = 1$ and $\mu = 2$ is presented in the same figure.

5 Conclusions

In this paper, we have presented an algorithm for the unavailable/missing samples reconstruction in the sparse signals. The algorithm is based on the concentration measures used to quantify the signal sparsity. Since the commonly used measures are not differentiable around the optimal point, a criterion for the variable algorithm parameters is introduced. The presented, gradient-based, adaptive step size, algorithm is able to achieve the computer precision accuracy in a simple and numerically efficient way. The algorithm is applied to the signals that are sparse in the DFT domain, including the signals that are only approximately sparse. An example with a noisy signal is considered. This algorithm can be applied on any concentration/sparsity measure form. A simple example on this topic is also presented.

The algorithm application may be extended to any non-stationary signal with an appropriately chosen transformation domain, where the considered non-stationary signal is sparse. The most obvious extension could be to the linearly frequency modulated signals that are sparse in the first-order polynomial Fourier transform domain. These signals are also sparse in the domain of the fractional Fourier transform. Further steps in the generalisation would be on the analysis of the higher order polynomial phase signals with the corresponding polynomial transforms, where these signals are sparse. Another possible extension of the presented algorithm would be on the analysis of the sinusoidally modulated signals, with the inverse Radon transform of their time-frequency representation as the sparsity domain.

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Biografija – Stefan Vujović

Stefan Vujović je rođen 1990. godine u Sarajevu, BiH. Dobitnik je diplome "Luča", a bio je i đak generacije u osnovnoj školi. Učestvovao je i ostvario zapažene rezultate na regionalnim i republičkim takmičenjima iz matematike i fizike za učenike osnovnih škola, a učestvovao je i na saveznom takmičenju iz fizike. Diplomirao je 2012, a magistrirao 2013. godine, oboje u oblasti elektrotehnike (digitalne obrade signala) na Univerzitetu Crne Gore. U tri navrata je dobio nagradu za najboljeg studente Elektrotehničkog fakulteta, kao i nagradu Univerziteta za najboljeg studenta Elektrotehničkog fakulteta. Nakon završetka magistarskih studija, zapošljava se kao saradnik i istraživač na Elektrotehničkom fakultetu. U naučno istraživačkom radu bavi se obradom signala i kompresivnim odabiranjem. Član je TFSA grupe na Elektrotehničkom fakultetu.

Nagrade, priznanja i stipendije:

- Nagrada Univerziteta Crne Gore za najboljeg studenta Elektrotehničkog fakulteta za prethodnu (2011) godinu.
- Nagrada Elektrotehničkog fakulteta za odlične rezultate i srednju prosječnu ocjenu A za prethodnu godinu, u tri navrata 2009, 2010 i 2011 godine
- Nagrada inženjerske komore Crne Gore za izuzetne rezultate tokom studija. Ovu nagradu dobija 5 najboljih studenata sa svih tehničkih fakultata
- Stipendija EPCG za 5 najboljih studenata Elektrotehničkog fakulteta
- Dobitnik stipendije Ministarstva prosvjete

Stefan je učestvovao u realizaciji brojnih naučno-istraživačkim projekata na elektrotehničkom fakultetu kao i na projektima Crnogorke akademije nauka i umjetnosti (CANU). Takođe je i recezent u brojnim renomiranim časopisima iz oblasti elektrotehnike.

Kompletna bibliografija – Stefan Vujović

Master teza:

1. **S. Vujović**, “Rekonstrukcija nedostajućih odbiraka signala upotrebom mjera koncentracije,” *M.S. Thesis*, Univerzitet Crne Gore, Podgorica, 2013

Vodeći naučni časopisi (SCI/SCIE lista):

1. **S. Vujović**, A. Draganić, M. Lakičević, I. Orović, M. Daković, M. Beko, and S. Stanković, “Sparse Analyzer Tool for Biomedical Signals,” *Sensors*, 20(9), 2602, doi: 10.3390/s20092602
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Drugi međunarodni, regionalni i nacionalni časopisi:

1. **S. Vujović**, M. Brajović, V. Popović-Bugarin, N. Latinović, J. Latinović, and M. Bajčeta, “A web service for grapevine monitoring and forecasting a disease,” *ETF Journal of Electrical Engineering*, Vol. 22, No. 1, 2016

Međunarodne konferencije:

1. M. Brajović, **S. Vujović**, I. Orović, and S. Stanković, “Coefficient Tresholding in the Gradient Reconstruction Algorithm for Signals Sparse in the Hermite Transform Basis,” *Applications of Intelligent Systems 2018 (APPIS 2018)*, Las Palmas De Gran Canaria, 8-12 January 2018
2. S. Stanković, **S. Vujović**, I. Orović, M. Daković, and LJ. Stanković, “Combination of Gradient Based and Single Iteration Reconstruction Algorithms for Sparse Signals,” *17th IEEE International Conference on Smart Technologies, IEEE EUROCON 2017*
3. **S. Vujović**, I. Stanković, M. Daković, and LJ. Stanković, “Comparison of a Gradient-Based and LASSO (ISTA) Algorithm for Sparse Signal Reconstruction,” *5th Mediterranean Conference on Embedded Computing MECO 2016*, Bar, June 2016

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5. **S. Vujović**, M. Daković, I. Orović, and S. Stanković, "An Architecture for Hardware Realization of Compressive Sensing Gradient Algorithm," *4th Mediterranean Conference on Embedded Computing, MECO – 2015*
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5. M. Daković, **S. Vujović**, and LJ. Stanković, "Rekonstrukcija odbiraka signala korišćenjem sparse signal analize," *Informacione Tehnologije - IT 2013*, Žabljak, februar 2013.

Prof. dr Irena Orović

BIOGRAFIJA

Irena Orović je rođena 21.02.1983.god. u Podgorici. Završila je studije na Elektrotehničkom fakultetu u Podgorici 2005. godine. Diplomirala je sa ocjenom 10 u Julu 2005. godine u Brestu, Francuska, gdje je boravila po osnovu bilateralne saradnje između Univerziteta Crne Gore i ENSIETA-e Brest. Od 2005-2010 godine bila je saradnik u nastavi na Elektrotehničkom fakultetu, zatim od 2010-2015 docent na Univerzitetu Crne Gore, od 2015-2020 vanredni profesor. U junu 2020. izabrana je u zvanje redovnog profesora.

Postdiplomske studije je upisala u septembru 2005. godine na Elektrotehničkom fakultetu (odsjek Elektronika, telekomunikacije i računari, smjer Računari).

Magistarsku tezu „**Primjena vremensko-frekvencijske analize na watermarking govornih signala**“ odbranila je sa ocjenom 10 u Decembru 2006. godine.

Doktorsku disertaciju: „**Vremensko-frekvencijske distribucije i neki aspekti primjene**“ odbranila je 19.02.2010. godine.

Dobitnik je brojnih nagrada i priznanja, među kojima treba istaknuti:

- Studentsku nagradu “19. decembar” (2003),
- Nagradu Crnogorske akademije nauka i umjetnosti (2004),
- Nagradu Univerziteta Crne Gore (2004),
- Više puta je nagrađivana od strane Elektrotehničkog fakulteta kao najbolji student generacije
- Dobitnik je Plakete Univerziteta Crne Gore za najboljeg diplomiranog studenta iz oblasti tehničkih, prirodno-matematičkih i medicinskih nauka (2005. godine),
- Dobitnik je nagrade Elektrotehničkog fakulteta za izvanredne naučno-istraživačke rezultate tokom rada na doktorskoj tezi (2010. godine).
- Dobitnik je internacionalne nagrade za najbolju doktorsku disertaciju TRIMO 2011 Ljubljana, Slovenija
- Nagrada Ministarstva nauke za najuspješniju ženu u nauci - 2012 godine

Boravci na inostranim naučnim institucijama: Dr. Orović je boravila na instituciji ENSIETA iz Bresta, Francuska (2005 i 2006.), University Bonn-Rhien-Sieg iz Bona, Njemačka (2007), Institut Polytechnique de *Grenoble*, Francuska (2008. i 2009.), Villanova University, Philadelphia USA (2010, 2011, 2012).

Prof. dr Irena Orović je do sada objavila oko 130 naučnih radova od čega oko 60 u vodećim svjetskim časopisima (časopisi sa SCI/SCIE liste sa impact faktorom), kao i veći broj radova u drugim međunarodnim časopisima i na konferencijama.

Objavila je kao koautor 5 udžbenika na našem jeziku. Od knjiga i monografija inostranih izdavača objavila je dvije knjige: “Multimedia Signals and Systems”, Springer 2012 na engleskom jeziku publikovanu od strane svjetskog izdavača Springer-a, kao i „Multimedia Signals and Systems: Basic and Advanced Algorithms for Signal Processing“, zatim poglavlje u međunarodnoj monografiji “Time-Frequency Analysis of Micro-Doppler Signals Based on Compressive Sensing,” Compressive Sensing for Urban Radar, CRC-Press, 2014”, poglavlje u enciklopediji: „Sparse Signal Reconstruction“ in Encyclopedia of Electrical and Electronics Engineering, Wiley 2017.

Recenzent je u mnogobrojnim časopisima, među kojima je više njih iz IEEE i IEE izdanja.

Bila je rukovodilac Računarskog centra na Elektrotehničkom fakultetu, i šef studijskog programa Elektronika, telekomunikacije, računari.

U periodu od 2011.-2015. godina dr Irena Orović je bila potpredsjednik i član Savjeta za naučno-istraživačku djelatnost u Crnoj Gori (Ministarstvo nauke Crne Gore).

Od decembra 2017. godine obavlja funkciju Prorektora za nauku i istraživanje.

Predsjednik je Naučnog odbora Univerziteta Crne Gore.

Skupština Crne Gore izabrala je u junu 2020. godine za člana nacionalnog Savjeta za visoko obrazovanje.

Više detalja i kompletan spisak referenci može se pronaći na sajtu www.tfsa.ac.me.

DESET ZNAČAJNIJIH REFERENCI

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2. **I. Orović**, and S. Stanković, "Improved Higher Order Robust Distributions based on Compressive Sensing Reconstruction," *IET Signal Processing*, 2014 (ISSN: 1751-9675, DOI: 10.1049/iet-spr.2013.0347)

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<http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6898675&newsearch=true&queryText=Improved%20Higher%20Order%20Robust%20Distributions%20based%20on%20Compressive%20Sensing%20Reconstruction>

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3. **I. Orović**, S. Stanković, and T. Thayaparan, "Time-Frequency Based Instantaneous Frequency Estimation of Sparse Signals from an Incomplete Set of Samples," *IET Signal Processing, Special Issue on Compressive Sensing and Robust Transforms*, 2014 (ISSN: 1751-9675, DOI: 10.1049/iet-spr.2013.0354)

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SCI lista:

<http://science.thomsonreuters.com/cgi-bin/jrnlst/jlresults.cgi?PC=K&Full=IET%20Signal%20Processing>

4. **I. Orović**, and S. Stanković, "L-statistics based Space/Spatial-Frequency Filtering of 2D signals in heavy tailed noise," *Signal Processing*, Volume 96, Part B, March 2014, Pages 190-202 (ISSN: 0165-1684, DOI: 10.1016/j.sigpro.2013.08.021)

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6. L.J. Stanković, **I. Orović**, S. Stanković, and M. Amin, "Compressive Sensing Based Separation of Nonstationary and Stationary Signals Overlapping in Time-Frequency," *IEEE Transactions on Signal Processing*, Vol. 61, no. 18, pp. 4562 – 4572, Sept. 2013. (ISSN: 1053-587X, DOI: 10.1109/TSP.2013.2271752)

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8. **I. Orović**, M. Orlandić, S. Stanković, and Z. Uskoković, "A Virtual Instrument for Time-Frequency Analysis of Signals with Highly Non-Stationary Instantaneous Frequency," *IEEE Transactions on Instrumentation and Measurement*, Vol. 60, No. 3, pp. 791 - 803, March 2011 (ISSN: 0018-9456, DOI: 10.1109/TIM.2010.2060227)

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10. S. Stanković, **I. Orović**, N. Žarić, and C. Ioana, "Two Dimensional Time-Frequency Analysis based Eigenvalue Decomposition Applied to Image Watermarking," *Multimedia Tools and Applications*, Vol.49, No. 3, Sept. 2010., pp. 529-543. (Print ISSN: 1380-7501, Online ISSN: 1573-7721, DOI: 10.1007/s11042-009-0446-x)

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Broj	05.06.2020		
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Na osnovu člana 72 stav 2 Zakona o visokom obrazovanju („Službeni list Crne Gore“ br 44/14, 47/15, 40/16, 42/17, 71/17, 55/18, 3/19, 17/19, 47/19) i člana 32 stav 1 tačka 9 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore na sjednici održanoj 04.06.2020. godine, donio je

ODLUKU O IZBORU U ZVANJE

Dr Irena Orović bira se u akademsko zvanje redovni profesor Univerziteta Crne Gore za **oblasti Računarstvo i Digitalna obrada signala**, na Elektrotehničkom fakultetu Univerziteta Crne Gore, na neodređeno vrijeme.



**SENAT UNIVERZITETA CRNE GORE
PREDSJEDNIK**

Prof. dr Danilo Nikolić, rektor

Prof. dr Miloš Daković

BIOGRAFIJA

Miloš Daković je rođen 1970. godine u Nikšiću, Crna Gora. Diplomirao je 1996., magistrirao 2001. i doktorirao 2005. godine, na Elektrotehničkom fakultetu Univerziteta Crne Gore. Redovni je profesor na Univerzitetu Crne Gore od 2017. godine.

Učestvovao je u više od 10 naučno-istraživačkih projekata finansiranih od strane Volkswagen fondacije, crnogorskog Ministarstva nauke i kanadske vlade (DRDC). Recenzent je u više međunarodnih časopisa, među kojima su: IEEE Transactions on Signal Processing, IEEE Signal Processing Letters, IEEE Transactions on Image Processing, IET Signal Processing, Signal processing i Geoscience and Remote Sensing Letters.

Dosadašnji naučno-istraživački rad profesora Dakovića rezultovao je objavljivanjem više od 100 radova, od čega je preko 40 u vodećim međunarodnim časopisima. Koautor je knjige *Time-Frequency Signal Analysis with Applications* čiji je izdavač Artech House, Boston.

Oblasti njegovog naučno-istraživačkog interesovanja su: obrada signala, vremensko-frekvencijska analiza signala, obrada radarskih signala i compressive sensing.

Dr Daković je dobitnik Godišnje nagrade za naučna dostignuća u 2015. godini, u kategoriji pronalazač – inovator za najuspješnije inovativno rješenje, koju uručuje Vlada Crne Gore.

Više detalja i kompletan spisak referenci može se pronaći na sajtu www.tfsa.ac.me.

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Link knjige na sajtu Amazon.com: <http://www.amazon.com/Time-Frequency-Signal-Analysis-Applications-Artech/dp/1608076512>
Pregled knjige dostupan je na books.google.com. Knjiga se može pronaći i na sajtu renomiranog međunarodnog izdavača Artech House: www.artechhouse.com
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5. **M. Daković**, T. Thayaparan, and LJ. Stanković, "Time-frequency based detection of fast manoeuvring targets," *IET Signal Processing*, Vol. 4, No. 3, June 2010, pp. 287-297. (ISSN: 1751-9675) DOI: 10.1049/iet-spr.2009.0078
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Link na rad: <http://ieeexplore.ieee.org/abstract/document/7126172/>
SCI lista:
<http://mjl.clarivate.com/cgi-bin/jrnlst/jlresults.cgi?PC=MASTER&ISSN=0018-9251>



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Broj / Ref: 03-79
Datum / Date: 12. 01. 2017

Na osnovu člana 72 stav 2 Zakona o visokom obrazovanju („Službeni list Crne Gore“ br. 44/14, 47/15, 40/16) i člana 32 stav 1 tačka 9 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore na sjednici održanoj 12. januara 2017. godine, donio je

ODLUKU O IZBORU U ZVANJE

Dr Miloš Daković bira se u akademsko zvanje redovni profesor Univerziteta Crne Gore za oblast Digitalna obrada signala i adaptivni sistemi na Elektrotehničkom fakultetu i na nematičnim fakultetima, na neodređeno vrijeme.

 **REKTOR**
Prof. Radmila Vojvodić

Crna Gora
UNIVERZITET CRNE GORE
ELEKTROTEHNIČKI FAKULTET

Primljeno: <u>17. 01. 2017</u>			
Org. jed.	Broj	Prilog	Vrijednost
<u>02/1</u>	<u>55</u>		

Doc. dr. sc. Jonatan Lerga

Jonatan Lerga Head of Department of Computer Engineering and Head of Laboratory for Application of Information Technologies with Faculty of Engineering, University of Rijeka, Croatia, received his PhD degree from the Faculty of Electrical Engineering and Computing, University of Zagreb, Croatia in 2011. Since 2007 he has been with the Faculty of Engineering, University of Rijeka, Croatia. In 2012 he received the annual award of the Croatian Academy of Engineering for his scientific achievements. He also received the annual award of the City of Rijeka in 2015 and the Primorje-Gorski Kotar County in 2018. Also, he received awards from the Foundation of the University of Rijeka in 2008, 2010 and 2018. His main research interests are statistical signal and image processing, time-frequency signal analysis, information theory, coding and signal processing applications.

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**Matični odbor za područje tehničkih znanosti
- polja elektrotehnike i računarstva**

KLASA: UP/I-640-03/18-01/0545
URBROJ: 355-06-04-18-0004
Zagreb, 12. srpnja 2018.

Sveučilište u Splitu	
FAKULTET ELEKTROTEHNIKE, STROJARSTVA I BRODOGRADNJE R.Boškovića 32, 21000 SPLIT	
Pril. br.: 24. 07. 2018	
Klasifikacijska oznaka:	Org. jed.
640-03/18-01/0001	
Uredžbeni broj:	Pril. i Vrij.
2181/206-03-03-18-0011	

Na temelju članka 35. i 35. Zakona o znanstvenoj djelatnosti i visokom obrazovanju (NN 123/03, 198/03, 105/04, 174/04, 46/07, 45/09, 63/11, 94/13, 139/13, 101/14, 60/15) Matični odbor za područje tehničkih znanosti – polje elektrotehnike i računarstva, na 5. sjednici održanoj 12. srpnja 2018. donosi

ODLUKU
o izboru u znanstveno zvanje

Dr. sc. JONATAN LERGA, docent Tehničkog fakulteta Sveučilišta u Rijeci, izabire se u znanstveno zvanje višeg znanstvenog suradnika u znanstvenom području tehničkih znanosti – polje računarstvo.

Obrazloženje

Sukladno članku 35. i 35. Zakona o znanstvenoj djelatnosti i visokom obrazovanju pristupnik dr. sc. Jonatan Lerga, podnio je dana 4. prosinca 2017. Fakultetu elektrotehnike, strojarstva i brodogradnje Sveučilišta u Splitu zahtjev za izbor u znanstveno zvanje višeg znanstvenog suradnika.

Na prijedlog Stručnog povjerenstva imenovanog na sjednici Fakultetskog vijeća Fakulteta elektrotehnike, strojarstva i brodogradnje Sveučilišta u Splitu dana 24. siječnja 2018., koje je za pristupnika dalo svoje mišljenje o ispunjenju uvjeta iz Pravilnika o uvjetima za izbor u znanstvena zvanja – čl. 15. tehničke znanosti (NN 28/17), Fakultetsko vijeće Fakulteta elektrotehnike, strojarstva i brodogradnje Sveučilišta u Splitu na svojoj sjednici održanoj 21. ožujka 2018. utvrdilo je da pristupnik ispunjava sve uvjete za izbor u znanstveno zvanje višeg znanstvenog suradnika u znanstvenom području tehničkih znanosti – polje računarstvo.

Matični odbor prihvatio je prijedlog Fakultetskog vijeća Fakulteta elektrotehnike, strojarstva i brodogradnje Sveučilišta u Splitu na 5. sjednici održanoj 12. srpnja 2018. te izabrao pristupnika u znanstveno zvanje višeg znanstvenog suradnika, uzevši u obzir čl. 32. st. 7. Zakona.

UPUTA O PRAVNOM LIJEKU: Protiv Odluke o izboru u znanstveno zvanje pristupnik nema pravo žalbe, ali može pokrenuti upravni spor pred Upravnim sudom u Rijeci u roku od 30 dana od dana dostave pristupniku. Tužba se predaje Upravnom sudu u Rijeci neposredno u pisanom obliku, usmeno na zapisnik ili se šalje poštom odnosno dostavlja elektronički.



Odluka se dostavlja:

1. Dr. sc. Jonatan Lerga
2. Fakultet elektrotehnike, strojarstva i brodogradnje u Splitu
3. Ministarstvo znanosti i obrazovanja



**Matični odbor za područje tehničkih znanosti
- polja elektrotehnike i računarstva**

KLASA: UP/I-640-03/19-01/0950

URBROJ: 355-06-04-19-0002

Zagreb, 12. srpnja 2019.

Na temelju članka 33. i 35. Zakona o znanstvenoj djelatnosti i visokom obrazovanju (NN 123/03, 198/03, 105/04, 174/04, 46/07, 45/09, 63/11, 94/13, 139/13, 101/14, 60/15) Matični odbor za područje tehničkih znanosti – polje elektrotehnike i računarstva, na 9. sjednici održanoj 12. srpnja 2019. donosi

ODLUKU
o izboru u znanstveno zvanje

Dr. sc. JONATAN LERGA, docent Tehničkog fakulteta Sveučilišta u Rijeci, izabire se u znanstveno zvanje višeg znanstvenog suradnika u znanstvenom području tehničkih znanosti – polje elektrotehnika.

Obrazloženje

Sukladno članku 33. i 35. Zakona o znanstvenoj djelatnosti i visokom obrazovanju pristupnik dr. sc. Jonatan Lerga, podnio je 16. svibnja 2019. Tehničkom fakultetu Sveučilišta u Rijeci zahtjev za izbor u znanstveno zvanje višeg znanstvenog suradnika.

Na prijedlog Stručnog povjerenstva imenovanog na sjednici Tehničkog fakulteta Sveučilišta u Rijeci 31. svibnja 2019., koje je za pristupnika dalo svoje mišljenje o ispunjenju uvjeta iz Pravilnika o uvjetima za izbor u znanstvena zvanja – čl. 1. tč. 2. tehničke znanosti (NN 84/05, 100/06, 138/06, 120/07, 71/10, 116/10, 38/11), Fakultetsko vijeće Tehničkog fakulteta Sveučilišta u Rijeci na svojoj sjednici održanoj 28. lipnja 2019. utvrdilo je da pristupnik ispunjava sve uvjete za izbor u znanstveno zvanje višeg znanstvenog suradnika u znanstvenom području tehničkih znanosti – polje elektrotehnika.

Matični odbor prihvatio je prijedlog Fakultetskog vijeća Tehničkog fakulteta Sveučilišta u Rijeci na 9. sjednici održanoj 12. srpnja 2019. te izabrao pristupnika u znanstveno zvanje višeg znanstvenog suradnika.

UPUTA O PRAVNOM LIJEKU: Protiv Odluke o izboru u znanstveno zvanje pristupnik nema pravo žalbe, ali može pokrenuti upravni spor pred Upravnim sudom u Rijeci u roku od 30 dana od dana dostave pristupniku. Tužba se predaje Upravnom sudu u Rijeci neposredno u pisanom obliku, usmeno na zapisnik ili se šalje poštom odnosno dostavlja elektronički.



Odluka se dostavlja:

1. Dr. sc. Jonatan Lerga
2. Tehnički fakultet u Rijeci
3. Ministarstvo znanosti i obrazovanja

Sveučilište u Rijeci
TEHNIČKI FAKULTET
Klasa: 120-01/19-01/34
Ur. broj: 2170-57-01-19-1
OIB: 46319717480
Rijeka, 19. 09. 2019.

Temeljem članka 31. Statuta Tehničkog fakulteta Sveučilišta u Rijeci dekanica donosi

ODLUKU O PLAĆI

- 1.) Dr. sc. **Jonatan Lerga** zaposlen je na položaju I. vrste, **predstojnik zavoda, docent** te mu se utvrđuje koeficijent složenosti poslova **2,037**.
- 2.) Zaposlenik-ca ima na dan donošenja ove Odluke ukupno 12 godina, 10 mjeseci i 1 dan radnog staža.
- 3.) Za svaku navršenu godinu radnog staža umnožak koeficijenta složenosti poslova i osnovice za izračun plaće uvećava se 0,5%.
- 4.) Zaposleniku-ci pripada dodatak na plaću od 15%, a za akademski stupanj doktora znanosti. Dodatak se obračunava dodavanjem na osnovnu bruto plaću.
- 5.) Ova Odluka stupa na snagu **01. listopada 2019. godine**, a primjenjuje se nakon što nadležno Ministarstvo odobri koeficijent.

Obrazloženje:

Primjenom članka 50. Temeljnog kolektivnog ugovora za službenike i namještenike u javnim službama («Narodne novine» br. 128/17) odlučeno je kao u izreci.
Pojmovi korišteni u ovoj Odluci koji imaju rodni značaj primjenjuju se jednako na osobe muškog i ženskog spola.

Pouka o pravnom lijeku:

Protiv ove Odluke može se dekanici uložiti zahtjev za zaštitu prava u roku od 15 dana od njenog primitka.

Dekanica:

Sveučilište u Rijeci
Prof. dr. sc. Jasna Prpić-Oršić
u.7.

Dostaviti:

1. Zaposlenik
2. Računovodstvo
3. Opća i kadrovska služba